Towards a model-free output tracking of switched nonlinear systems[1]

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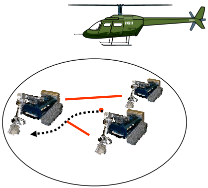
Robocoop http://syner.ec-lille.fr/robocoop/

OUTLINE

Introduction
Switched systems

☞ Switchings between several subsystems (DS_i) : controller design (switching supervisory control, ...) or inherently (Robocoop project : http ://syner.ec-lille.fr/robocoop)



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☞ Existing results need a complete description of all DS_i

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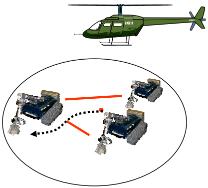

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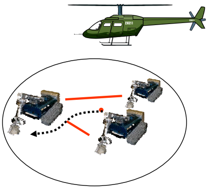

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Main idea

☞ Here the control problem (output tracking) is tackled without a complete description of the subdynamics and even without knowing the switching signal

☞ Switched systems (without state jumps) = collection of differential algebraic i/o relation (locally : implicit theorem)

$$\boxed{y^{(p)} = a(.) + b(.)u}$$

$a(.)$ and $b(.)$ may change with the switching signal.

☞ Fast online estimations of $a(.), b(.), y^{(i)}, i = 1, \ldots, p-1$ $\Rightarrow$ desired tracking performances using GPID, PID or "state" feedback, for example (if $b(.) \neq 0$)

$$[b(.)]_{\text{estim}}\, u + [a(.)]_{\text{estim}} + \sum_{i=-1}^{(p-1)} \alpha_i \left(\left[y^{(i)} \right]_{\text{estim}} - y_{\text{ref}}^{(i)} \right) = y_{\text{ref}}^{(p)}$$

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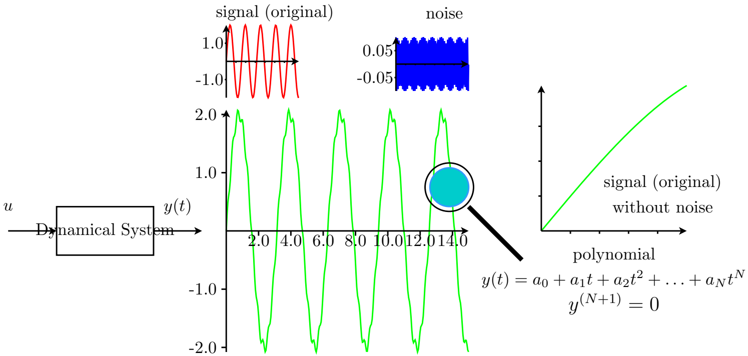

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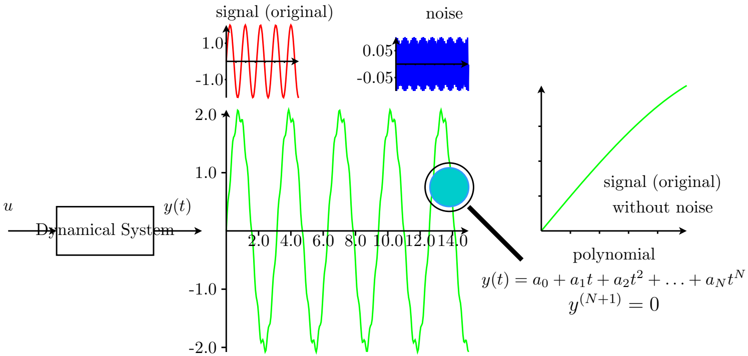

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Example Cruise control

$$\dot{v} = -\frac{\beta v^2}{M}\mathsf{sgn}(v) - g\sin(\alpha(t)) + \frac{T(i)}{M}u,$$

- output v : speed,
- M : mass of the vehicle,
- $\alpha(t)$: road incline,
- $T(i)$ motor torque (gear $i \in \{1, 2, 3, 4, 5\}$),
- $a(.) = -\frac{\beta v^2}{M}\mathrm{sgn}(v) - g\sin(\alpha(t))$ is usually not well known,
- $b(.) = \frac{T(i)}{M}$ is changing with the gear selected by the driver.

☞ Construct a control law such that v follows a given trajectory v_{ref}.

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☞ (Sampling) $T_s << T_{\text{window}}$ length of the sliding window : small enough w.r.t the variation of $a(.), b(.)$ ($\approx$ constants $a_{0_{\text{estim}}}, b_{0_{\text{estim}}}$).

☞ On $[t, t + T_{\text{window}}[$: $\dot{y} = a_0 + b_0 u$

☞ fast estimates of $\dot{y} = \dot{v}$ (or a_0) and b_0 :

$[\dot{v}]_{\text{estim}} - b_{0_{\text{estim}}} u(kT_s) \approx a_{0_{\text{estim}}}$

☞ Sampled control :

$$u((k+1)T_s) = -\frac{(k_p e_y + k_i \int e_y) - (([\dot{v}]_{\text{estim}} - \dot{v}_{\text{ref}}) - b_{0_{\text{estim}}} u(kT_s))}{b_{0_{\text{estim}}}},$$

where $e_y = v - v_{\text{ref}}$ is the relative error for y, ones get that

$$\ddot{e}_y + k_p \dot{e}_y + k_i e_y \simeq 0, \text{ in fact } o(T_s)$$

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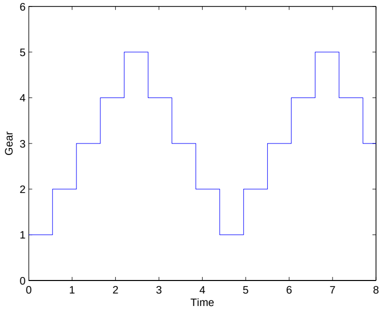
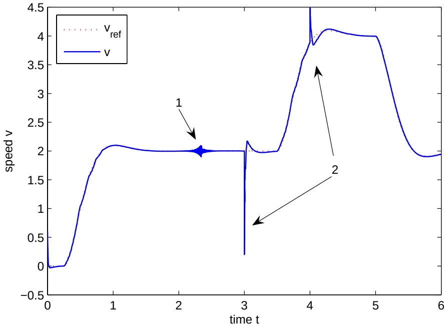


Switching signal



Trajectory of the switched system 2

Introduction
Example Cruise control

Derivative estimates :

$$y(t) = y(0) + y'(0)t, \quad (\mathcal{L}(s))$$

$$Y(s) = \frac{y(0)}{s} + \frac{y'(0)}{s^2} \quad (\times s)$$

$$sY(s) = y(0) + \frac{y'(0)}{s} \quad (D_s)$$

$$Y(s) + s\frac{dY}{ds}(s) = -\frac{y'(0)}{s^2} \quad (\times s^{-3})$$

$$y'(0) = \frac{\int_0^t \left[24(t-\tau) - 12(t-\tau)^2\right] y(\tau)d\tau}{t^4}$$

(see the paper for more details)

Introduction

Example Cruise control

Parameter estimation : On $[t, t+T_{\text{window}}[$: $\dot{y} = a_0 + b_0 u$

$$\dot{y} = a_0 + b_0 u, \quad (\mathcal{L}(s))$$

$$sY(s) - y_0 = \frac{a_0}{s} + b_0 U(s), \quad (s^2 \ D_s)$$

$$s^2 Y(s) + s^3 \frac{d}{ds} Y(s) = -a_0 + b_0 s^2 \frac{d}{ds} U(s) \quad \mathcal{L}^{-1}(s^{-3} \ D_s)$$

$$b_0 = \frac{\int_0^t P(t,\tau) y(\tau) d\tau}{\int_0^t Q(t,\tau) u(\tau) d\tau},$$

$$P(t,\tau) = -(t-\tau)^2 + 4\tau(t-\tau) - \tau^2,$$

$$Q(t,\tau) = -\tau(t-\tau)^2 + \tau^2(t-\tau).$$

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Parameter estimation : On $[t, t + T_{\text{window}}[$: $\dot{y} = a_0 + b_0 u$

$$s^2 Y(s) + s^3 \frac{d}{ds} Y(s) = -a_0 + b_0 s^2 \frac{d}{ds} U(s), \quad (\times s^{-3})$$

$$s^{-2} Y(s) + s^{-1} \frac{d}{ds} Y(s) = -\frac{a_0}{s^4} + b_0 s^{-2} \frac{d}{ds} U(s), \quad (\mathcal{L}^{-1})$$

$$a_0 = \frac{b_0 \int_0^t R(t,\tau) u(\tau) d\tau + \int_0^t S(t,\tau) y(\tau) d\tau}{t^3},$$

$$R(t,\tau) = -6\tau(t-\tau), S(t,\tau) = -6(t-2\tau).$$

(see the paper for more details and general formulae)

Singularity arise in b_0 estimate when the stabilization is obtained since both terms of the quotient will be zero.

Introduction
ALIEN impacts

ALIEN's techniques proceed :

- they are algebraic : operations on s-functions ;
- they are non-asymptotic (formal expressions using ratios of integrals) ;
- they are deterministic : no knowledge of the statistical properties of the noise n is required.

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ALIEN impacts : Control Applications

- **Closed loop identification** : Real time parameter identification (in closed loop while the plant is working even with time delay).
- **State reconstructors** : Classical technics (for LTI sys.) :
  - asymptotic observers (sensitive to mismatches and perturbations),
  - Kalman filters (Riccati equation and precise statistics of the noise),

  For nonlinear systems the question has remained largely open in spite of a huge literature (ALIEN solution : D_t estimate).
- **Blind control design**.
- **Fault diagnosis** detecting and isolating a fault in closed loop for a possibly uncertain system remains largely open.

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ALIEN impacts : Applications to signal, image and video processing

Three patents are already pending in those topics :

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Control design
Problem formulation

Nonlinear switched systems into input/output form :

$$0 = f_{\sigma(t)}(t, y, \dot{y}, \ldots, y^{(p_{\sigma(t)})}, u, \ldots, u^{(m_{\sigma(t)})}, d).$$

- $\sigma : \mathbb{R} \to I = \{1, \ldots, N\}, t \mapsto \sigma(t)$ switching signal,
- $u \in \mathbb{R}$ input,
- $y \in \mathbb{R}$ measured output,
- d disturbances
- $x \in \mathbb{R}^{n_{\sigma(t)}}$ state of variable dimension : $n_{\sigma(t)}$.

Assumption : NMPh Subsystems (zero dynamics AS).
Problem : For a large class of such systems, for any $\sigma(t)$ with a minimal given activation time $T_{\text{min}}^{\text{active}}$ and only using the measured output (noisy) we want the output to track a given signal.

Control design

Outline of the control design procedure

Main Assumption 1 :

$\exists p \in \{1, \ldots, \min_{i \in I}(p_i)\}$ such that (at least locally) we have

$$y^{(p)} = a_{\sigma(t)}(.) + b_{\sigma(t)}(.)u \qquad (1)$$

where the functions $a_{\sigma(t)}(.)$ and $b_{\sigma(t)}(.)$ depend on $(t, y, \ldots, y^{(p_{\sigma(t)})}, u, \ldots, u^{(m_{\sigma(t)})}, d)$. From now, if fast estimations of $a_{\sigma(t)}$ and $b_{\sigma(t)}$ are available then the tracking problem is solved using control :

$$[b(.)]_{\text{estim}}\, u + [a(.)]_{\text{estim}} + \sum_{i=-1}^{(p-1)} \alpha_i \left(\left[y^{(i)} \right]_{\text{estim}} - y^{(i)}_{\text{ref}} \right) = y^{(p)}_{\text{ref}}$$

Control design

Outline of the control design procedure

Already shown on the cruise control example (for general case see paper) :

1. there exists "good" realtime estimation of time derivative of noisy signal,
2. there exists "good" realtime estimation of $a_{\sigma(t)}$ and $b_{\sigma(t)}$ using only the noisy output and the input.

It remains to show that :

1. there exists a "large" class of switched systems that meet Assumption 1,
2. the closed-loop system is UAS (w.r.t. σ).

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Control design
Comments on Assumption 1

$$y^{(p)} = a_{\sigma(t)}(.) + b_{\sigma(t)}(.)u$$

☞ exact knowledge of $b_{\sigma(t)}$ is not required since $a_{\sigma(t)}(.)$ may contain u : what is needed is an order of this term.
Criteria : subsystems of explicit form with b_i :

1. of the same order : then we **will not** estimate this parameter,
2. varying with the active dynamics then we **will** estimate this parameter online (sufficiently fast).

Numerical implementation : the first case it is much more simple.

Control design

Comments on Assumption 1

First case, the required conditions are :

1. $\forall i \in I : \exists P_i = \left\{ p_{i,j} \in \{1, \ldots, p_i\} \subset \mathbb{N} | \frac{\partial f_i}{\partial y^{(p_{i,j})}} \neq 0 \right\}$. Set $p = \min(\cap_i P_i)$ this is the smallest integer so that $\frac{\partial f_i}{\partial y^{(p)}} \neq 0, \forall i \in I$. Implicit theorem (at least locally) :

$$y^{(p)} = F_{\sigma(t)}(t, y, \ldots, y^{(p-1)}, y^{(p+1)}, \ldots, y^{(n_{\sigma(t)})}, u, d). \qquad (2)$$

2. $\forall i \in I : \left. \frac{\partial f_i}{\partial u} \right|_{u=0} \neq 0$, then obtain from the physics or some experiments a rough range estimate of $\alpha_i = \left. \frac{\partial F_i}{\partial u} \right|_{u=0} : \alpha_{\sigma(t)} \in [m, M] : \alpha_{\sigma(t)}$ is of order $10^o, o \in \mathbb{N}$. Lastly, rewrite (2) as

$$y^{(p)} = a_{\sigma(t)}(.) + 10^o u. \qquad (3)$$

Control design
Comments on Assumption 1

Second case, the required conditions are :

1. $\forall i \in I$: Let $p \in \{1, \ldots, p_i\}$ be the smallest integer so that $\frac{\partial f_i}{\partial y^{(p)}} \neq 0, \forall i \in I$. Implicit theorem (2) holds (at least locally).
2. $\forall i \in I$: $\left.\frac{\partial f_i}{\partial u}\right|_{u=0} \neq 0$, then rewrite (2) as

$$y^{(p)} = a_{\sigma(t)}(.) + b_{\sigma(t)}(.)u. \qquad (4)$$

☞ In practice $p = 1$ or 2 is sufficient (most of the time) : let us stress that it **does not imply** that the system should be of order one or two.

Control design

Numerical implementation of the control

☞ T_s : sampling period thus the discrete version of (1) should be

$$[b(.)]_{\text{estim}}(kT_s)u(kT_s) = -[a(.)]_{\text{estim}}(kT_s) + y_{\text{ref}}^{(p)}(kT_s) - \sum_{i=-1}^{(p-1)} \alpha_i \left(\left[y^{(i)} \right]_{\text{estim}}(kT_s) - y_{\text{ref}}^{(i)}(kT_s) \right). \quad (5)$$

☞ time sliding window of length $T_{\text{window}} = \kappa T_s$ [2] used to estimate $[a(.)]_{\text{estim}}$, $[b(.)]_{\text{estim}}$ and $\left[y^{(i)} \right]_{\text{estim}}$ by using only the successive values of the measured output and input it implies that we have (up to some small error) the following errors estimates :

[2] In prcatice $\kappa \in [100, 300]$

Control design

Numerical implementation of the control

First case using (3) :

$$\left[y^{(p)}(kT_s)\right]_{\text{estim}} + o(T_s) \approxeq y^{(p)}(kT_s) = a_{\sigma(t)}(.)\Big|_{(kT_s)} + 10^o u(kT_s)$$

Thus $[a(.)]_{\text{estim}}(kT_s)$ can be obtained directly using

$$[a(.)]_{\text{estim}}(kT_s) \approxeq \left[y^{(p)}(kT_s)\right]_{\text{estim}} - 10^o u(kT_s).$$

But if we plug this estimate into (5) an algebraic loop appears, thus since $[a(.)]_{\text{estim}}(kT_s) \approxeq [a(.)]_{\text{estim}}((k-1)T_s)$, one gets the control $u(kT_s)$ for $t \in [kT_s, (k+1)T_s[$ defined by

$$10^o\,[u(kT_s) - u((k-1)\,T_s)] = -\left[y^{(p)}\right]_{\text{estim}}\Big|_{((k-1)T_s)} - \sum_{i=-1}^{p-1} \alpha_i \left(\left[y^{(i)}\right]_{\text{estim}} - y^{(i)}_{\text{ref}}\right) + y^{(p)}_{\text{ref}} \Big| kT_s$$

Control design

Numerical implementation of the control

In (2) we get

$$y^{(p)}(t) - y_{\text{ref}}^{(p)}\Big|_{(kT_s)} = \left[a_{\sigma(t)}(.)(t) - [a(.)]_{\text{estim}}|_{(kT_s)}\right] - \sum_{i=-1}^{p-1} \alpha_i \left(\left[y^{(i)}\right]_{\text{estim}} - y_{\text{ref}}^{(i)}\right) + y_{\text{ref}}^{(p)}\Big|_{kT_s} .$$

Thus using reasonable assumptions (regularity of $a_{\sigma(t)}$)

$$\left| a_{\sigma(t)}(.)(t) - [a(.)]_{\text{estim}}|_{(kT_s)} \right| = o(T_s) \qquad (6)$$

as soon as there is no switching time in between this time interval.
Now, from the time derivative estimates :

$$\left| \left[y^{(i)} \right]_{\text{estim}} (kT_s) - y^{(i)}(t) \right| = o(T_s) \qquad (7)$$

leading to $e_y^{(p)} + \sum_{i=-1}^{p-1} \alpha_i e_y^{(i)} = o(T_s)$, ($e_y = y(t) - y_{\text{ref}}(t)$), for $t \in [kT_s, (k+1)T_s[$ as soon as there is no switching time in between this time interval.

Control design

Numerical implementation of the control

Second case using (4) :

Similar argument leads to a similar estimation. implementation of the control

Control design

Closed-loop practical stability

By adjusting the coefficients α_i it is possible to adjust the time response t_r : clearly after a total time of $T_{\text{window}} + t_r$ the response error is near to zero. Adapting a well known results ([?][?]) about asymptotic stability of switched asymptotically stable system for any switching law having a given dwell time it is clear that we need

$$T_{\text{window}} + t_r < T_{\text{min}}^{\text{active}}. \quad (8)$$

to guarantee uniform asymptotic stability of the closed-loop system for any switching signal having a minimum time of activation $T_{\text{min}}^{\text{active}}$. Note that t_r can be tuned using the α_i and $T_{\text{window}} = \kappa T_s$. Thus (8) give a rule to select the sampling period knowing only the minimal time of activation of a subsystem :

$$T_s < \frac{T_{\text{min}}^{\text{active}} - t_r}{\kappa}. \quad (9)$$

Examples

$$\dot{x} = f_i(x) + g_i(x)u + d, \quad x \in \mathbb{R}^{n_i}$$
$$y = h_i(x, u) + n,$$

n : noise, d : disturbance

i	$f_i(x)$	$g_i(x)$	$h_i(x,u)$	n
1	$-x$	1	x	1
2	$2x$	1	x	1
3	$\begin{pmatrix} 0 & -3 & -3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$x_1 + 3x_2 + x_3$	3
4	$\begin{pmatrix} 2 & -3 \\ 1 & 0 \end{pmatrix} x$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \end{pmatrix} x$	2
5	$\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} x$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \end{pmatrix} x$	2
6	$5x + 10\sin(x)$	1	x	1
7	$-2x + 10\exp(x)$	1	x	1

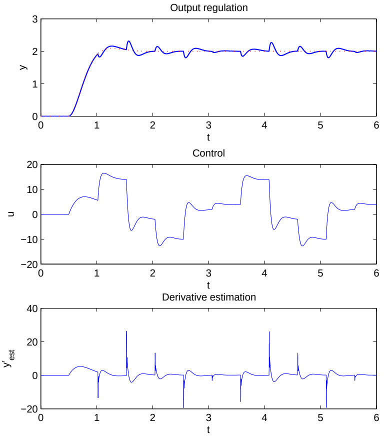
linear stable ($i = 1$), unstable ($i = 2, 3, 4$), non minimum phase ($i = 5$), or even nonlinear ($i = 6, 7$).

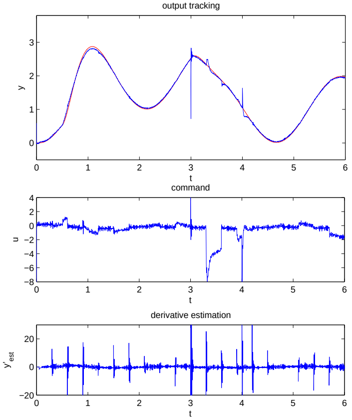
Switching signal is randomly selecting any subsystem every 0.5 s (thus here $T_{\text{min}}^{\text{active}} = 0.5$ s).

$t_r = 0.4$ s $\Rightarrow \kappa = 100, T_s = 0.001$ s, $T_{\text{window}} = \kappa T_s$ =0.1 s.

Examples

b_i are of the same magnitude : case 1

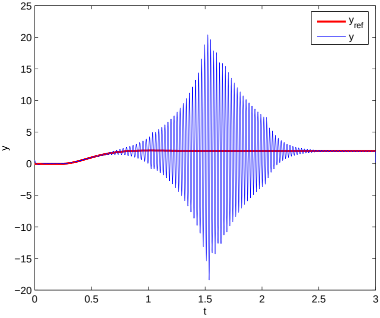


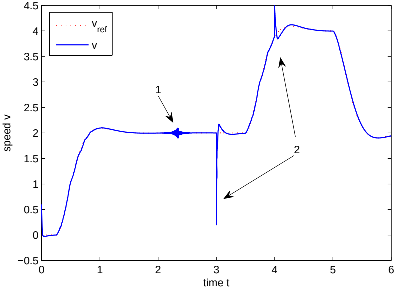


Nonlinear case with noise and

Examples
Cruise control

Introductive example, the coefficient $\frac{T(i)}{M}$ takes its values among $\{1, 5, 10, 30, 50\}$





With estimation.

Without an estimation of b

Conclusion

New approach for hybrid systems control, without any description of the subdynamics :

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3. time between two switchings is large enough

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