An Iterative and Joint Estimation of SNR and Frequency Selective Channel for OFDM Systems

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1. Background
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   - Estimation Methods

2. Proposed Method
   - Presentation of the Algorithm
   - Convergence of the Algorithm

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1. Background
   System Model
   In the frequency domain, the \( n^{th} \) received OFDM symbol is:

\[
\mathbf{U}_n = \mathbf{C}_n \mathbf{H}_n + \mathbf{W}_n
\]

with
- \( \mathbf{U}_n \): Vector of the received OFDM symbol
- \( \mathbf{C}_n \): Matrix of the emitted OFDM symbol
- \( \mathbf{H}_n \): Vector of the frequency channel response
- \( \mathbf{W}_n \): AWGN
- \( H_{m,n} = \sum_{l=0}^{L-1} h_{l,n} \exp (-2j\pi \frac{m}{M} \tau_l) \)

where
- \( m = 0, 1, ..., M - 1 \)
- \( h_{l,n} \): zero mean Gaussian process
- \( L \): number of paths of the channel
- \( \tau_l \): \( l^{th} \) path delay
1. Background

Estimation Methods - Signal to noise ratio (SNR, noted $\rho$)

\[ \hat{\rho} = \frac{\hat{\mu}^2}{\sigma^2} - 1 \]
with $\mu_2$ the second moment-order of the received signal

- [1]: \( \hat{\sigma}^2 \) estimated thanks to the subtraction of two consecutive received signal vectors
  \[ \hat{\sigma}^2 = \frac{1}{2}||\textbf{W}_n - \textbf{W}_{n-1}||^2 = \frac{1}{2}||\textbf{R}_n - \textbf{R}_{n-1}||^2 \]

- [2]: \( \hat{\sigma}^2 \) estimated thanks to the subspace properties of the estimated covariance matrix of the received signal

Our solution: \( \hat{\sigma}^2 \) estimated thanks to the MMSE criterion

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1. Background

Estimation Methods - Noise estimation

- MMSE criterion performed on the pilot (index $p$)

\[ \hat{\sigma}^2 = \frac{1}{M}E\{|\textbf{U}_p - \textbf{C}_p \hat{\textbf{H}}_p|^2\} \]

- In practice: approximation of MMSE criterion

\[ \hat{\sigma}^2 = \frac{1}{M} \sum_{m=0}^{M-1} |\textbf{U}_{m,p} - \textbf{C}_{m,p} \hat{\textbf{H}}_{m,p}|^2 \]

The noise variance estimation depends on the channel estimation quality
1. Background
Estimation Methods - Channel estimation

- Least square (LS) estimator:
  \[ \hat{H}_p^{LS} = C_p^{-1} U_p = H_p + C_p^{-1} W_p \]

- Linear minimum mean square error (LMMSE) estimator:
  \[ \hat{H}_p^{LMMSE} = R_H (R_H + \sigma^2 (C_p C_p^H)^{-1})^{-1} \hat{H}_p^{LS}, \]
  with \( R_H \) the frequency covariance matrix of the channel

- For the noise variance estimation:
  - LS estimator: not adapted for the noise variance estimation
    \[ \hat{\sigma}^2 = \frac{1}{M} \sum_{m=0}^{M-1} |U_{m,p} - C_{m,p} \hat{H}_p^{LS}|^2 = 0 \]
  - LMMSE estimator: efficient estimator, adapted for the noise variance estimation
    requires nevertheless an estimation of the noise variance

2. Proposed Method
Presentation of the Algorithm

We suppose pilots as \( C_p = I \)

Problem: \( H_p^{LMMSE} \) requires \( \hat{\sigma}^2 \), and \( \hat{\sigma}^2 \) requires \( H_p^{LMMSE} \)

Solution: an iterative algorithm

At each iteration \( i \), we do
\[
\begin{align*}
H_p^{LMMSE}(i) &= R_H (R_H + \hat{\sigma}^2(i-1) I)^{-1} \hat{H}_p^{LS} \\
\hat{\sigma}^2(i) &= \frac{1}{M} \sum_{m=0}^{M-1} |U_{m,p} - C_{m,p} H_p^{LMMSE}(i)|^2
\end{align*}
\]

(\( \hat{\sigma}, H_p^{LMMSE} \))
2. Proposed Method

Presentation of the Algorithm

- Initialization of the Algorithm:

If $\hat{\sigma}^2(0) = 0$, the LMMSE estimation is equivalent to the LS one:

$$\hat{H}_{p(m)}^{LMMSE} = \mathbf{R}_f((\mathbf{R}_f + \sigma^2 F_{i-1})^{-1} \mathbf{H}_p^{LS}$$

$$= \mathbf{R}_f((\mathbf{R}_f + \sigma^2 F_{i-1})^{-1} \mathbf{H}_p^{LS}$$

$$= \mathbf{H}_p^{LS}$$

The initialization is chosen so that $\hat{\sigma}^2(0) > 0$

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2. Proposed Method

Convergence of the Algorithm - noise variance

- From the theoretical expression of the MMSE noise variance estimation:

$$\hat{\sigma}_n^2 = \frac{1}{M} E\{||\hat{H}_{p(m)}^L - \hat{H}_{p(m)}^{LMMSE}||^2\}$$

- After some mathematical developments:

$$\hat{\sigma}_n^2 = f(\hat{\sigma}_n^2) = \frac{\hat{\sigma}_n^2}{\lambda_1 M} \sum_{m=0}^{M-1} \lambda_m \hat{\sigma}_n^2$$

- Convergence by solving $f(x) = x$ with

$$f(x) = \frac{\hat{\sigma}_n^2}{M} \sum_{m=0}^{M-1} \lambda_m + \sigma^2$$

Solution: the fixed point theorem
2. Proposed Method

Convergence of the Algorithm

Solution: the fixed point theorem applied to the function $f$

\[ f\left(\left[\frac{1}{M} \sigma^2, M_2\right]\right) \subset \left[\frac{1}{M} \sigma^2, M_2\right] \]

$\quad f' > 0$, so $f$ is strictly growing

$\implies f$ has at least one fixed point

\[ \hat{\sigma}_\text{est}^2 \text{ is upper and lower bounded: } \frac{1}{M} \sigma^2 \leq \hat{\sigma}_\text{est}^2 \leq M_2 \]

with $L$ the number of paths of the channel

$\hat{\sigma}_\text{est}^2$ is monotonous

$\implies \hat{\sigma}_\text{est}^2$ converges to a fixed point of $f$

Unicity of convergence soon published

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2. Proposed Method

Convergence of the Algorithm - channel estimation

Thanks to

\[ \hat{\mathbf{H}}_\text{LMMSE}^{L(i)} = \mathbf{R}_H (\mathbf{R}_H + \hat{\sigma}_\text{est}^2 D)^{-1} \hat{\mathbf{H}}_\text{LS}^{L(i)} \]

$\hat{\sigma}_\text{est}^2$ converges $\implies \hat{\mathbf{H}}_\text{LMMSE}^{L(i)}$ converges

What about the speed of convergence and the bias of the estimator?
### 3. Simulation Results

**Case 1:** Perfect covariance matrix: \( \mathbf{R}_H = \mathbf{H}_p \mathbf{H}_p^H \)

**Case 2:** Approximate covariance matrix:

\[
(\mathbf{R}_H)_{i,j} = \sum_{l=0}^{L-1} \sum_{k=0}^{M-1} \gamma_k^l \gamma_j^l e^{-j2\pi(\eta_i - \eta_j)}
\]

**Convergence of the algorithm**

- High speed of convergence: \( \leq 3 \) iterations
- Low bias: <2\% for SNR=0 dB and <5 \% for SNR=10 dB

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**Comparison of SNR estimation with other methods**

- Ren’s method: requires 2 pilots by preamble
- Xu’s method: requires 1 pilot by preamble
- Our method: requires 1 pilot by preamble

**Case 1:** perfect covariance matrix

**Case 2:** approximation of the covariance matrix

Good trade-off between efficiency and number of pilot required for the estimation
3. Simulation Results

Channel estimation

Proof by simulation of the convergence of the channel estimation

Gap between perfect estimation and Case 2 <0.5 dB (<0.1 dB in Case 1)

4. Conclusion

• New algorithm for joint estimation of SNR and multipath channel

• Proof of convergence of the algorithm

• Good quality of channel and SNR estimations with high speed of convergence

• Improvement of the trade-off between the number of required pilots and quality of estimation, compared with existing methods in literature

• Further works and publications:
  - Unicity of convergence of the algorithm
  - Development of a practical solution with an estimated frequency channel covariance matrix $R_{yy} = H_yL_yH_y^H$
Thank you for your attention

Questions ? ...