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# PAPR Frequency View for Cognitive Radio Systems

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**SUPELEC**

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Seminar SCEE

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Seminar SCEE

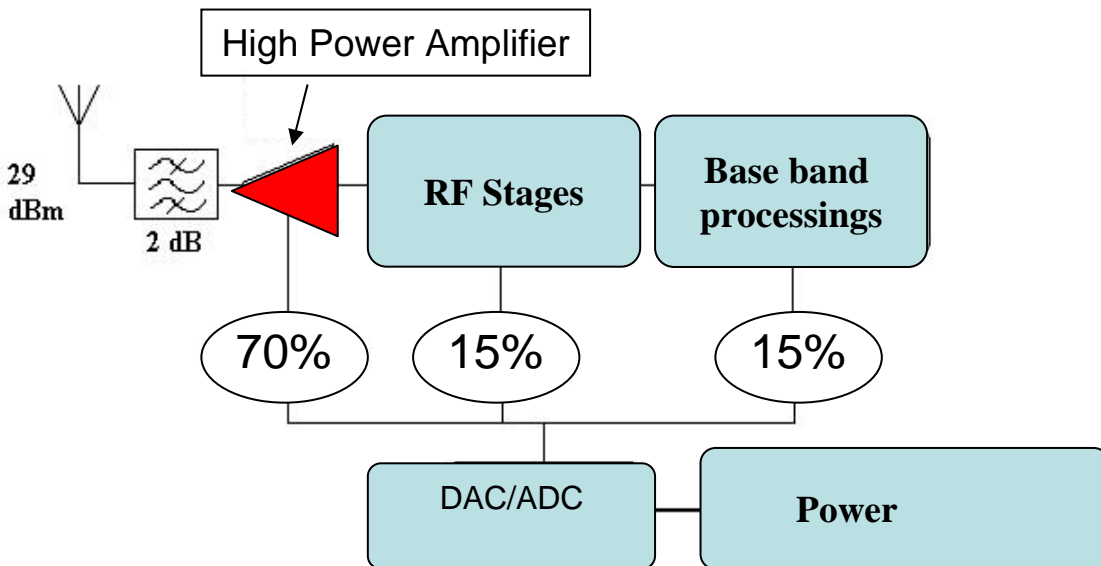
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- Non-linearities and PAPR Problem in Cognitive Radio (CR) systems
  - OFDM and Software Radio signal equivalence
  - Cognitive Radio systems : Dynamic Spectrum Access
- PAPR Frequency Domain Interpretation in CR
  - Free spectrum access under PAPR metric constraint
  - PAPR reduction methods based on adding signals in the frequency domain (tone reservation, ...)

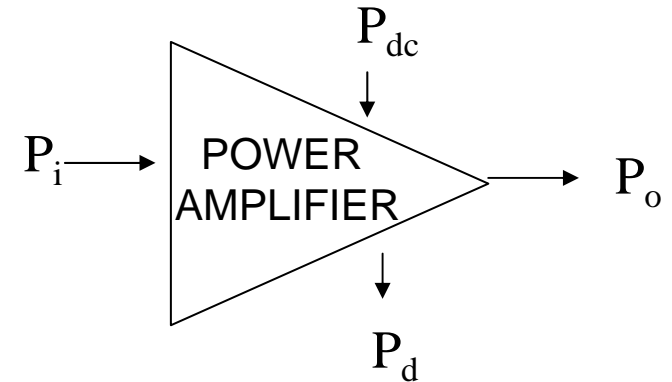
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## A 2.5G terminal power budget:

- Power : A key factor in telecommunications
- 60-70 % of power consumption due to HPA\*



\* HPA : High Power Amplifier



Power budget :

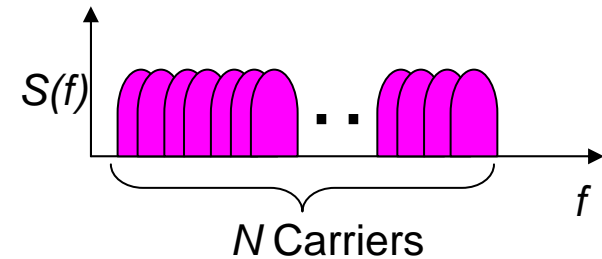
$$P_i + P_{dc} = P_o + P_d$$

Power efficiency :  $\eta = \frac{P_o}{P_{dc}}$

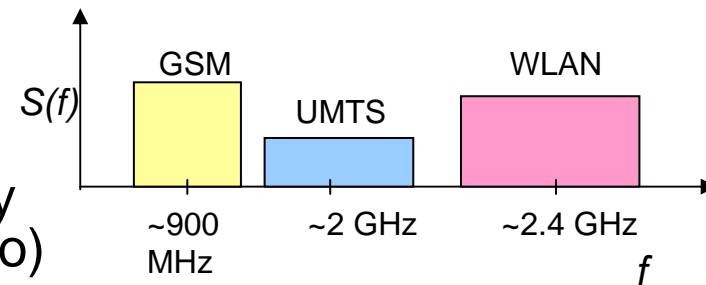


- OFDM is a multiplex of modulated carriers.
  - Multicarrier nature of OFDM causes large power fluctuations.
- SWR is also a multiplex of modulated carriers
  - Case 1 : Mono Standard signals ( $\Delta f = cnst$ )
  - Case 2 : Multi Standard signals ( $\Delta f \neq cnst$ )
 => SWR signal also demonstrates large power fluctuations.
- Power fluctuations are generally described by the term PAPR (Peak to Average Power Ratio)
  - PAPR=maximum power/ average power
- As PAPR is a random variable so it is presented by its cumulative distribution function,

$$CCDF = Pr[PR > PR_0]$$



OFDM frequency view



SWR frequency view

For signals with large number of modulated sub-carriers  $N$ , we have shown the equivalence between OFDM and SWR PAPR distribution.

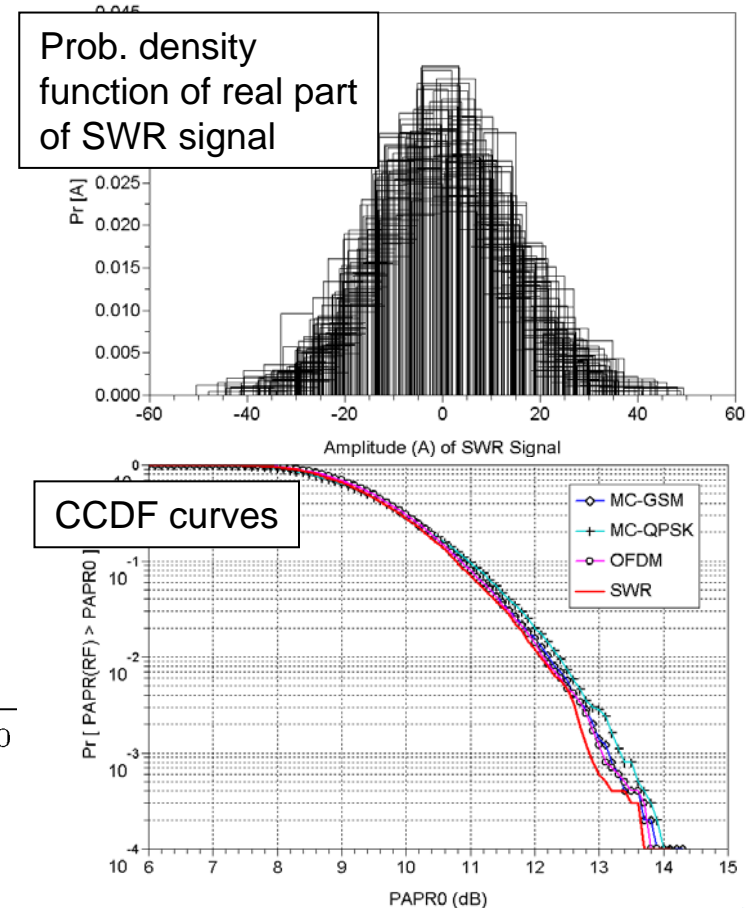
SWR=MC-GSM+ MC-QPSK+OFDM  
with  $N=64$   $L=4$  for all modulations

SWR signal follows classical OFDM PAPR distribution curve:

$$Pr[PR \geq PR_0] \approx 1 - e^{-\sqrt{\frac{\pi}{6}} N \sqrt{PR_0} e^{-PR_0}}$$

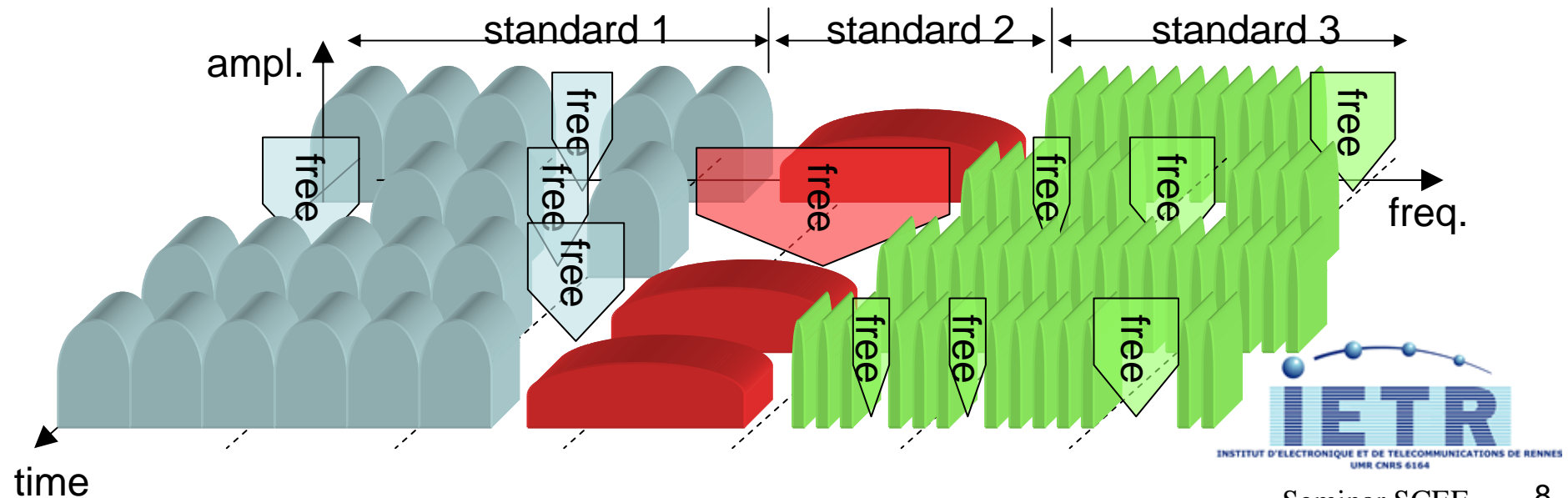
Conclusions :

Equivalence between OFDM and SWR motivated to use same analysis and the same methods for PAPR mitigation as for OFDM.

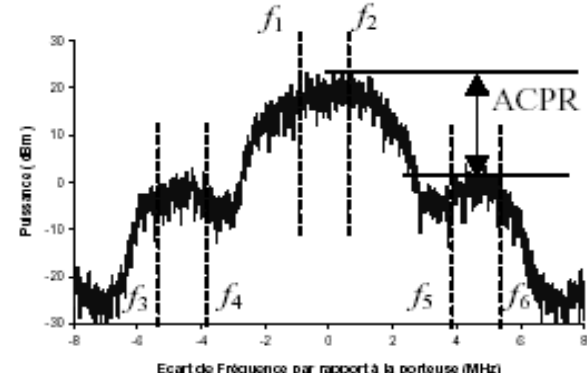
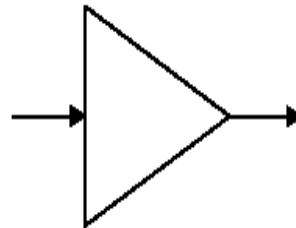
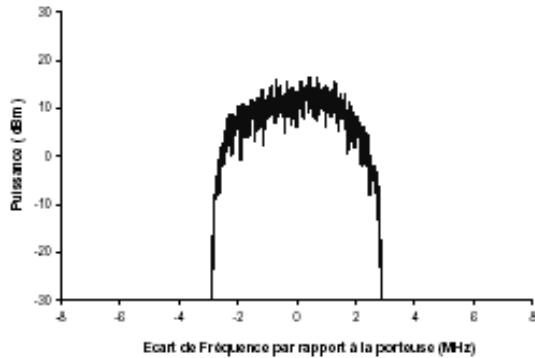


# Cognitive Radio and Dynamic Spectrum Access

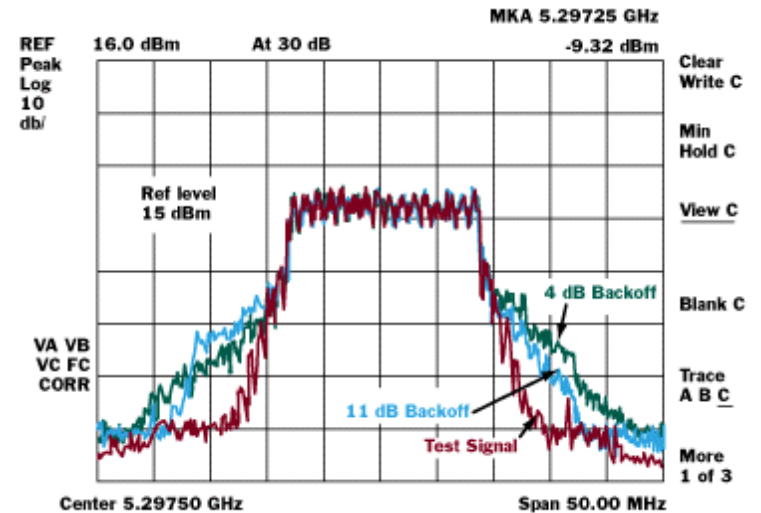
- Real time modification of the physical layer parameters (link adaptation).
  - Insert spectrum (tones) in between standards or inside the standards (opportunistic communication).
    - Find hole, check if useful and then insert the tones
- => *Global PAPR is modified*



# Cognitive Radio and Dynamic Spectrum Access

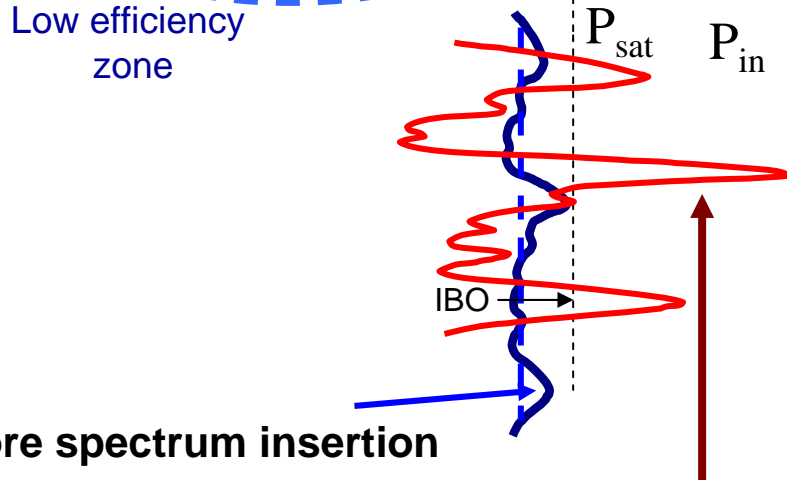
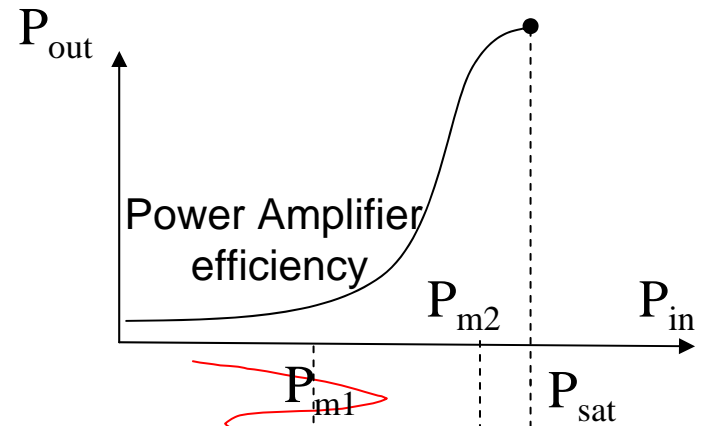
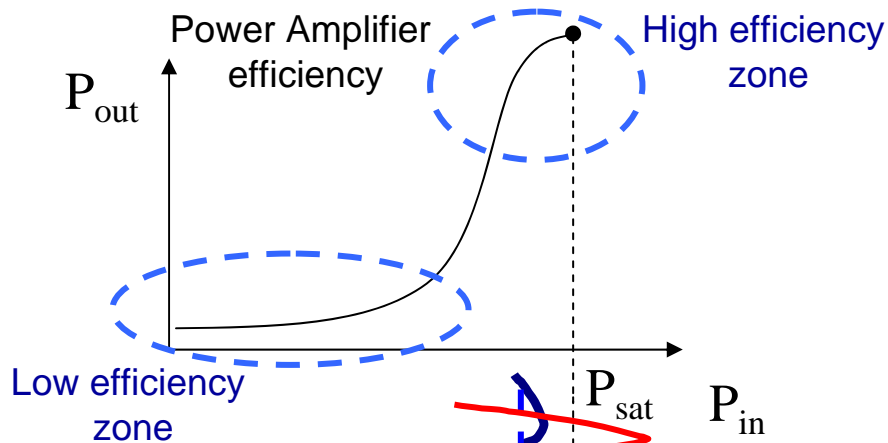


$$ACPR_{dB} = 10 \cdot \log \frac{\int_{f_1}^{f_2} P(f) df}{\int_{f_3}^{f_4} P(f) df + \int_{f_5}^{f_6} P(f) df}$$

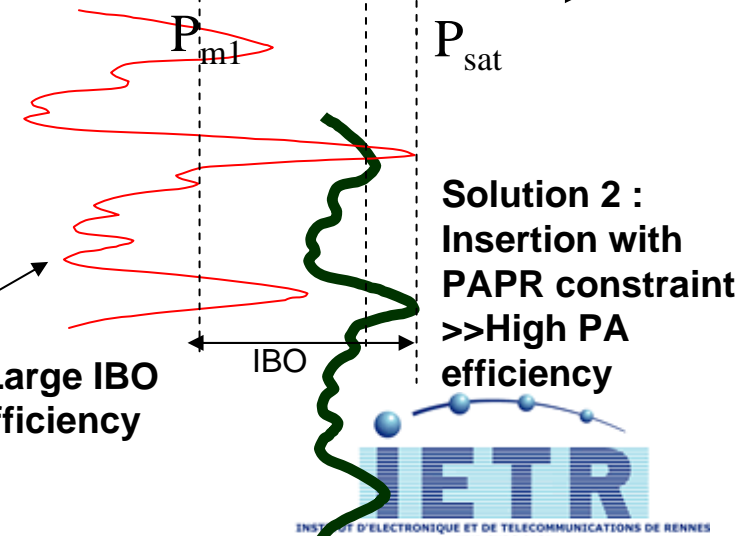


**Spectrum access should not increase the out of band power level to avoid interference with other signals**

- PAPR mitigation solutions after spectrum access



Solution 1 : Large IBO  
>> Low PA efficiency

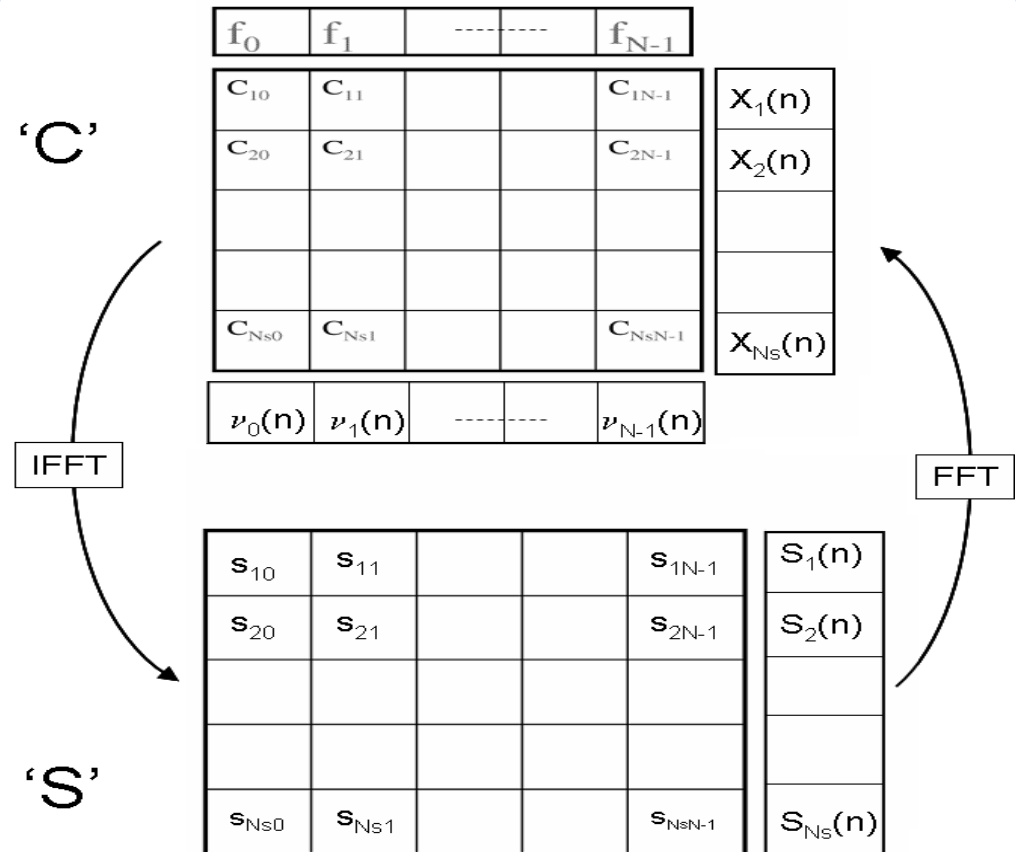


after spectrum insertion w/o taking PAPR into account

- Non-linearities and PAPR problem in Cognitive Radio
  - OFDM and Software Radio signals equivalency
  - Cognitive Radio systems : dynamical Spectrum Access
- PAPR Frequency Domain Interpretation
  - Free spectrum access under PAPR metric constraint
  - PAPR reduction methods based on adding signals in the frequency domain (tone reservation, ...)

# Frequency Domain PAPR Interpretation (1/7)

- In OFDM frequency symbols 'C' are tabulated as carrier per carrier.
- If  $N_s$  (Number of OFDM symbols) increases, PAPR approaches theoretical upper bound.
- Rows and columns interchangeable for PAPR calculations.



$$\sum_{r=1}^{N_s} \sum_{c=1}^N C_{c,r} \exp(i2\pi rc/N) = \sum_{c=1}^N \sum_{r=1}^{N_s} C_{c,r} \exp(i2\pi rc/N)$$

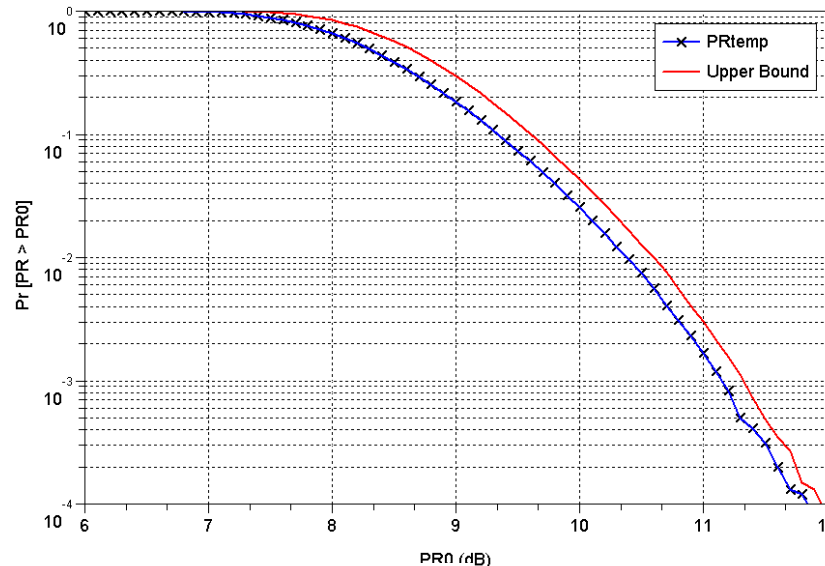
Goal : Express (time) PAPR vs individual carrier PAPR as each carrier PAPR being very easy to compute

$$PR_{N_s}(S(n)) \leq \frac{(\sum_{k=1}^N P_m(k) \times PR_{f_k}) + \max_{k \in K} (\max_{j \in J} (\lambda(j,k)))}{\sum_{k=1}^N P_m(k)} \quad (1)$$

where

$$\lambda(j, k) = \sum_{p=1}^N C_{j,p} \sum_{p' \neq p} \overline{C_{j,p'}} e^{2i\pi \frac{k(p-p')}{N}}, j \in J, k \in K.$$

- It demonstrates that  $PR_{temp}$  is upperbounded by relation (1) which depends only on frequency components.

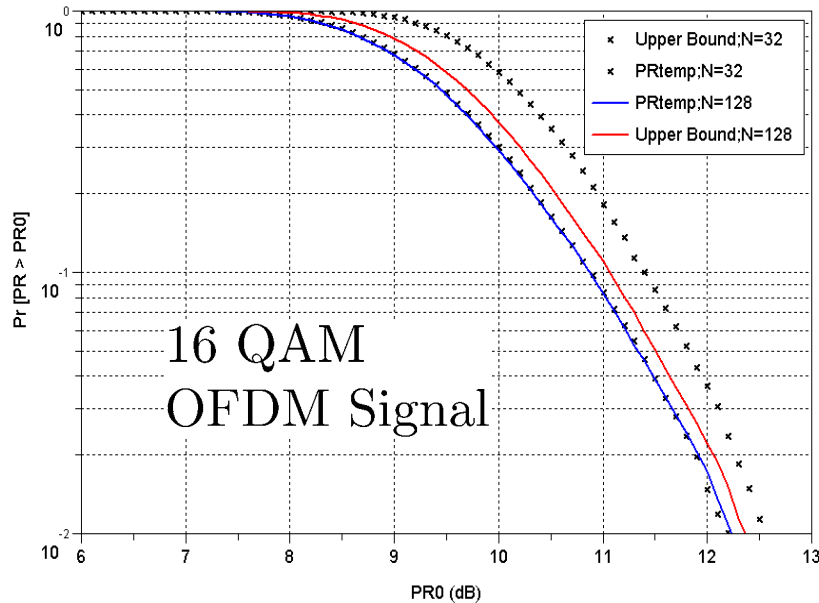


Upperbound and  $PR_{temp}$  distribution for OFDM

16 QAM  
64 Carrier  
OFDM Signal

- FFT size and modulation type effects

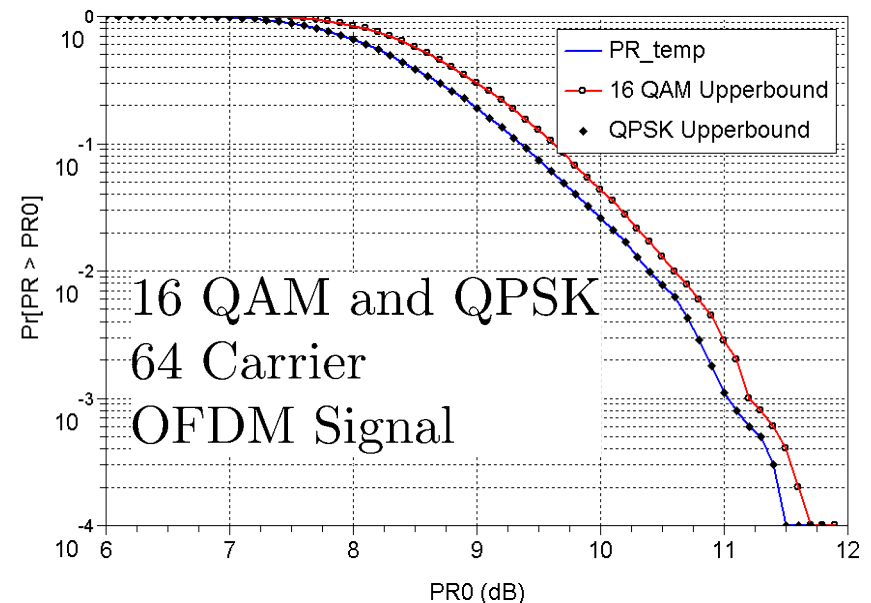
FFT size effect on OFDM upper bound



16 QAM  
OFDM Signal

Upperbound approaches  $PR_{temp}$  when N becomes large

Modulation type effect on OFDM upper bound



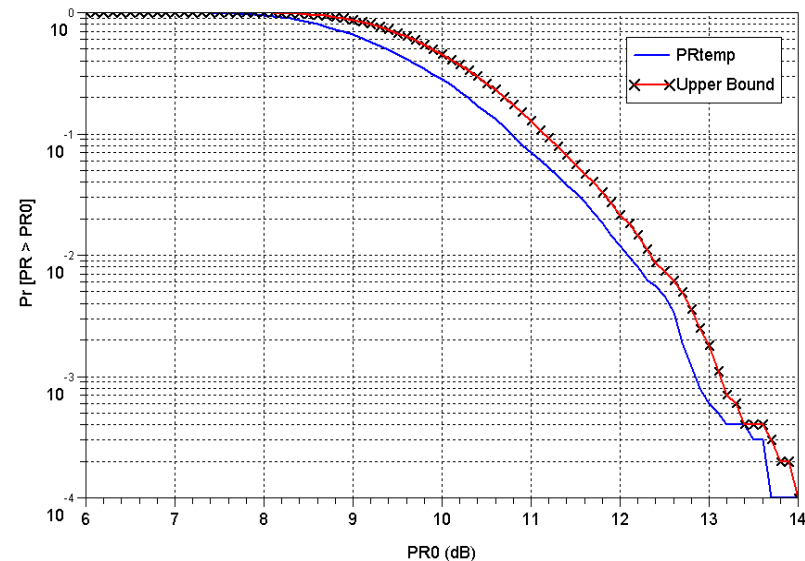
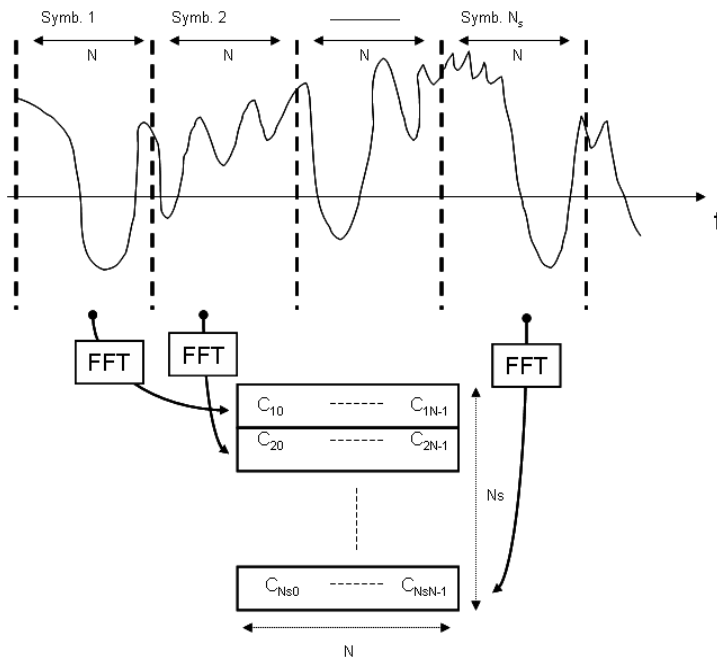
16 QAM and QPSK  
64 Carrier  
OFDM Signal

Upperbound =  $PR_{temp}$  for PSK modulations



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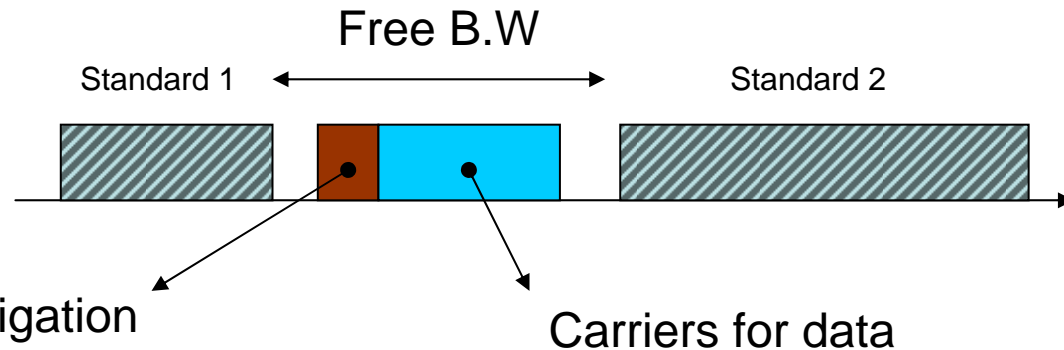
- Application to any SWR signals
  - SWR signal is sliced into pieces of FFT size
  - FFT is taken on these pieces to get 'C'
  - Eq. (1) is applied on 'C' and upper bound is obtained



SWR=MC-GMSK+ MC-QPSK+OFDM  
 with N=64 L=4

-With the help of this view we can :

- Estimate the spectrum access influence on global PAPR with the help of individual frequential PAPR
- Optimize some of these added carriers for PAPR mitigation (while the others to transmit useful data)



Possible methods : TR, Geometrical tone adding method <sup>(1)</sup>

<sup>(1)</sup> D Guel, J Palicot, Y Louët, "A Geometric Method for PAPR Reduction in a Adding Signal in context for OFDM Signals", International Conference on Digital Signal Processing, DSP 2007, 1-5 July 2007, Cardiff, UK

## • Added Carrier Effect on PAPR Upper Bound

$$PAPR_{N_s}(S(n)) \leq PAPR_{init} + PAPR_{added}$$

$$PAPR_{init} = PAPR_p$$

$$PAPR_{added} = PAPR_s + PAPR_m$$

with

$$PAPR_p \approx \frac{\max_{k \in K} (\max_{j \in J} (|\sum_{p \in P} C_{j,p} e^{i2\pi kp/N}|^2))}{\sum_{k=1}^N P_m(k)}$$

$$PAPR_s \approx \frac{\max_{k \in K} (\max_{j \in J} (|\sum_{s \in S} C_{j,s} e^{i2\pi ks/N}|^2))}{\sum_{k=1}^N P_m(k)}$$

$$PAPR_m \approx \frac{\max_{k \in K} (\max_{j \in J} (2|\sum_{p \in P} C_{j,p} e^{i2\pi kp/N}| |\sum_{s \in S} C_{j,s} e^{i2\pi ks/N}|))}{\sum_{k=1}^N P_m(k)}$$

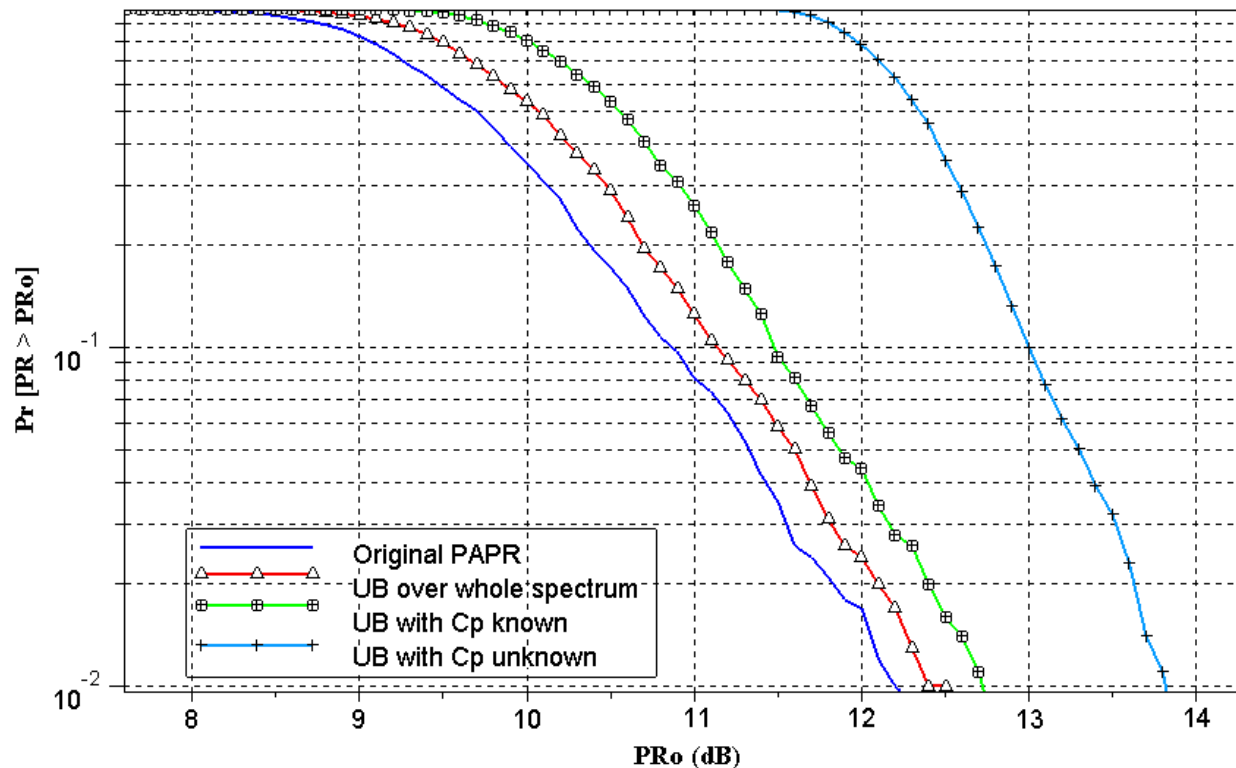
where

$p \in P$  : Primary carriers

$s \in S$  : Secondary carriers

- With limited spectral knowledge of primary user:

$$PAPR_{added} \leq \frac{\max_{k \in K} (\max_{j \in J} (|\sum_{s \in S} C_{j,s} e^{i2\pi ks/N}|^2 + 2 \cdot N_p \cdot |C_{p,max}| \sum_{s \in S} C_{j,s} e^{i2\pi ks/N}|))}{\sum_{k=1}^N P_m(k)}$$



$N_p = 62$   
 $N_s = 4$  QPSK symbols

- Spectrum access in CR should be:
  - performed taking care of the amplification problems
- PAPR frequency domain view would help in:
  - Free spectrum access under PAPR metric constraint.
  - Application of PAPR reduction methods based on adding signals in the frequency domain (tone reservation, ...).

# Thank you !!!

# Questions???