ENHANCED SPECTRUM SENSING TECHNIQUES FOR COGNITIVE RADIO SYSTEMS

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Outline

- Part 1: FFT and Filter Bank Based Spectrum Sensing for WLAN Signals
- Part 2: Optimized FFT and Filter Bank Based Spectrum Sensing for Bluetooth Signal
- Part 3 A: FFT and Filter Bank Based Spectrum Sensing and Spectrum Utilization for Cognitive Radio
- Part III B: Spectrum Sensing And Spectrum Utilization Model For OFDM and FBMC Based Cognitive Radios
- Part 4 A: Performance Analysis of Eigenvalue Based Spectrum Sensing under Frequency Selective Channels
- Part 4 B: Reducing Computational Complexity of Eigenvalue Based Spectrum Sensing for Cognitive Radio
- Part 4 C: Reduced Complexity Spectrum Sensing Based on Maximum Eigenvalue and Energy
Part 1 : FFT and Filter Bank Based Spectrum Sensing for WLAN Signals
INTRODUCTION

- **2.4 GHz ISM Band**
  - Due to the global availability, low cost wireless systems use this band
  - One important problem is interferences to each other

- **Cognitive Radio**
  - Potential solution for interference problem
  - Spectral holes can be determined robustly

- **Spectrum Sensing**
  - One of the main tasks of a CR for non-interfered spectrum
  - Different spectrum sensing algorithms can be applied such as blind and non-blind
OFDM and Filter Bank Multicarrier (FBMC)
- Effects in terms of spectrum sensing
- FFT and AFB based spectrum sensing

The goal of this paper
- Basic spectrum sensing functions for a CR in the ISM band
- CP-OFDM based 802.11g WLANs
- Most basic spectrum sensing method, energy detection
- FFT and AFB in analyzing the radio scene consisting of WLAN signals at different center frequencies
AFB AND FFT BASED ENERGY DETECTOR ALGORITHMS

Block diagram of energy detector with AFB and FFT based spectrum analysis

- The block diagram of alternative FFT and AFB based spectrum sensing algorithms is shown in Figure.
AFB AND FFT BASED ENERGY DETECTOR ALGORITHMS

Typically, the sampling rate of the subbands is equal to the ADC sampling rate divided by the number of channels in filter bank. In the spectrum sensing context, the subband signals can be expressed as

\[ Y(m,k) = \begin{cases} 
W(m,k) & H_0 \\
S(m,k)H_k + W(m,k)H_1 & H_1 
\end{cases} \]

- where \( S(m,k) \) is the transmitted signal by primary users as it appears at the \( m^{th} \) FFT or AFB output sample in subband \( k \), and \( W(m,k) \) is the corresponding channel noise sample. \( H_0 \) and \( H_1 \) illustrate the absent hypothesis and present hypothesis of a primary user (PU), respectively.

- In this study, rectangular filter window is used for its simplicity and lower computational complexity. In this case, the decision statistics at different frequencies can be calculated as

\[ \tilde{Y}(m,k) = \sum_{l=k-L_f/2}^{k+L_f/2-1} \sum_{u=m-L_y+1}^{m} |Y(u,l)|^2 \]
As $Y(k)$ has Gaussian distribution, the decision statistics $\tilde{Y}(k)$ follows chi-square distribution with degrees of freedom, $2L_fL_t$. Hence, can be modeled as

$$\tilde{Y}(k) = \begin{cases} \frac{\sigma^2_w}{2} \chi^2_{2L_fL_t} & H_0 \\ \frac{\sigma^2_w + \sigma_k^2}{2} \chi^2_{2L_fL_t} & H_1, \quad k \in \kappa \end{cases}$$

When the signal is present, $P_D$ is considered and thus it can be defined as

$$P_D = \Pr(\tilde{Y} > \lambda \mid H_1)$$

When the signal is absent and there is only noise, but the decision device decides incorrectly that a signal is present, the false alarm probability $P_{FA}$ is formulated as

$$P_{FA} = \Pr(\tilde{Y} > \lambda \mid H_0)$$

With the above assumptions, the probabilities $P_{FA}$ and $P_D$ can be written as

$$P_{FA} = 1 - \Gamma(L_fL_t, \frac{\lambda}{\sigma_w^2})$$

$$P_D = 1 - \Gamma(L_fL_t, \frac{\lambda}{\sigma_w^2 + \sigma_k^2})$$
We can see the ideal OFDM signal spectrum, which has a deep hole in the considered 3 MHz frequency band.

In the worst case situation allowed by the 802.11g specifications, the power spectral density in the gap can be at about -20 dBr (20 dB below the passband level) in the considered case.

Figure shows also a third case with modest spectral regrowth at -30 dBr level.
Actual false alarm probability of WLAN signals with target $P_{FA}=0.1$ for 3 MHz sensing bandwidth in AWGN case.

Figure shows the actual false alarm probabilities as a function of the active WLAN SNRs for AWGN.
Actual false alarm probability of WLAN signals with target $P_{FA} = 0.01$ for 3 MHz sensing bandwidth in Rayleigh case.

Figure show the actual false alarm probabilities as a function of the active WLAN SNRs for Rayleigh type frequency selective channels.
CONCLUSION

- FFT or filter band based spectrum analysis methods has been studied in this paper.
- WLAN signals are characterized by rather limited spectral purity, which results in significant interference in the possible spectral gaps between WLAN signals.
- This leads to obvious hidden node problems when trying to use the WLAN spectral gaps in a well-controlled way for secondary transmissions.
- With worst-case energy leakage (at about 20 dBr level) of WLAN signals to the spectral gaps, filter bank based spectrum analysis doesn’t provide significant benefits over FFT-based methods.
- However, already 10 dB reduction in the spectral regrowth level reveals the spectral leakage problems of FFT and AFB methods become clearly better.
Part 2 : Optimized FFT and Filter Bank Based Spectrum Sensing for Bluetooth Signal
INTRODUCTION

- **OFDM Based WLAN and FHSS Based Bluetooth**
  - Bluetooth sensing is analyzed also in the presence of WLANs at nearby frequencies.

- **The goal of this paper**
  - Energy detector based spectrum sensing techniques are optimized for detecting Bluetooth signals, considering both the effect of non-flat power spectrum and frequency hopping characteristics.
  - To reduce complexity and required number of samples for effective spectrum sensing, optimum weighting process is proposed for subband based spectrum sensing.
Optimizing Energy Detection for Non-Flat PU Spectrum

Block diagram of energy detector with AFB and FFT based spectrum analysis

• The block diagram of alternative FFT and AFB based spectrum sensing algorithms is shown in Figure.

• Weight process is applied after the time filtering process to reduce the complexity/required # of samples.
OPTIMIZING ENERGY DETECTION FOR NON-FLAT PU SPECTRUM

\[
Y(m,k) = \begin{cases} 
W(m,k) & H_0 \\
S(m,k)H_k + W(m,k) & H_1 
\end{cases}
\]

\[
T_K = \sum_{k=K-L_f}^{K+L_f} w_k T_k = \sum_{k=K-L_f}^{K+L_f} w_k \left( \frac{1}{L_t} \sum_{m=1}^{L_t} |Y(m,k)|^2 \right)
\]

\[
f(T_k)\big|_{H_0} \sim N\left(\sigma_n^2, \frac{1}{L_t}\sigma_n^4\right)
\]

\[
f(T_k)\big|_{H_1} \sim N\left(\sigma_k^2 + \sigma_n^2, \frac{1}{L_t}(\sigma_k^2 + \sigma_n^2)^2\right)
\]

\[
f(T_K)\big|_{H_0} \sim N\left(\sum_{k=K-L_f}^{K+L_f} w_k^2 \sigma_n^2, \frac{1}{L_t} \sum_{k=K-L_f}^{K+L_f} w_k^4 \sigma_n^4\right)
\]

\[
f(T_K)\big|_{H_1} \sim N\left(\sum_{k=K-L_f}^{K+L_f} w_k^2 (\sigma_k^2 + \sigma_n^2), \frac{1}{L_t} \sum_{k=K-L_f}^{K+L_f} w_k^4 (\sigma_k^2 + \sigma_n^2)^2\right)
\]
**Optimizing Energy Detection for Non-Flat PU Spectrum**

\[
P_D = \Pr(T > \lambda \mid H_1) \quad P_{FA} = \Pr(T_K > \lambda \mid H_0)
\]

\[
\lambda = Q^{-1}(P_{FA}) \sqrt{\frac{1}{L_t} \sum_{k=K-L_f}^{K+L_f} w_k^4 \sigma_n^4} + \sum_{k=K-L_f}^{K+L_f} w_k^2 \sigma_n^2
\]

\[
P_{FA} = Q\left(\frac{\lambda - \sum_{k=K-L_f}^{K+L_f} w_k^2 \sigma_n^2}{\sqrt{\frac{1}{L_t} \sum_{k=K-L_f}^{K+L_f} w_k^4 \sigma_n^4}}\right)
\]

\[
\lambda = Q^{-1}(P_D) \sqrt{\frac{1}{L_t} \sum_{k=K-L_f}^{K+L_f} w_k^4 \left(\sigma_n^2(1 + \text{SNR}_k)\right)^2} + \sum_{k=K-L_f}^{K+L_f} w_k^2 \sigma_n^2 \left(1 + \text{SNR}_k\right)
\]

\[
P_D = Q\left(\frac{\lambda - \sum_{k=K-L_f}^{K+L_f} w_k^2 \left(\sigma_n^2 + \sigma_k^2\right)}{\sqrt{\frac{1}{L_t} \sum_{k=K-L_f}^{K+L_f} w_k^4 \left(\sigma_n^2 + \sigma_k^2\right)^2}}\right)
\]

\[
W_k^2 = \frac{1}{K+L_f} \sum_{k=K-L_f}^{K+L_f} \frac{\sigma_k^2}{\sigma_n^2 \text{SNR}_k^2}
\]

\[
L_t = \frac{\sum_{k=K-L_f}^{K+L_f} w_k^4 \sigma_n^4 - Q^{-1}(P_{FA}) \sqrt{\sum_{k=K-L_f}^{K+L_f} w_k^4 \left(\sigma_n^2(1 + \text{SNR}_k)\right)^2}}{\left[ \sum_{k=K-L_f}^{K+L_f} w_k^2 \sigma_n^2 \text{SNR}_k^2 \right]^2}
\]
The Frequency Hopped Frequency Shift Keying (FH-FSK)-based 802.15 Bluetooth signal has 79 different frequency channels at center frequencies starting from 2.402 GHz and ending at 2.480 GHz, with 1 MHz spacing. The nominal bandwidth of BT signal is 1 MHz and the hopping rate is 1600 hops/sec. We consider first a simplified scheme with continuous BT signal at the 33rd channel.

Bluesooth signal spectrum in 2.4 GHz ISM band
Simulation Results of FFT and AFB Based Energy Detector Spectrum Sensing Algorithms

<table>
<thead>
<tr>
<th>Bluetooth SNR</th>
<th>Weight Factors</th>
<th>11 subbands</th>
<th>5 subbands</th>
<th>3 subbands</th>
<th>1 subband</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>Const.</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Opt.</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>-3 dB</td>
<td>Const.</td>
<td>39</td>
<td>25</td>
<td>23</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Opt.</td>
<td>21</td>
<td>21</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>-4 dB</td>
<td>Const.</td>
<td>60</td>
<td>37</td>
<td>34</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Opt.</td>
<td>31</td>
<td>31</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>-5 dB</td>
<td>Const.</td>
<td>92</td>
<td>56</td>
<td>51</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>Opt.</td>
<td>45</td>
<td>46</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>-6 dB</td>
<td>Const.</td>
<td>143</td>
<td>84</td>
<td>77</td>
<td>132</td>
</tr>
<tr>
<td></td>
<td>Opt.</td>
<td>68</td>
<td>69</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>-7 dB</td>
<td>Const.</td>
<td>223</td>
<td>131</td>
<td>117</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>Opt.</td>
<td>104</td>
<td>105</td>
<td>111</td>
<td></td>
</tr>
</tbody>
</table>

- We can see that almost the same time record length can be used when sensing a single subband at the BT center frequency as when sensing the whole 1 MHz BT band.

- With constant weights, 3 subbands is the optimum choice in all these cases.

- Using optimum weights, the sensing time can be reduced by about 10 %, and most of this benefit is gained by using only 5 subbands.

Required time record length in different weighting schemes
Two WLAN channels and Bluetooth signal spectra in 2.4 GHz ISM band. (a) WLAN channels 3 and 8, (b) WLAN channels 3 and 7.

- In the worst case situation allowed by the 802.11g specifications, the power spectral density in the 3 MHz gap can be at about -20 dBr (20 dB below the passband level) in the considered case.
- The figure shows also a third case with modest spectral regrowth at -30 dBr level.
- In the ‘no gap’ case, there isn’t really any spectral hole.
Simulation Results of FFT and AFB Based Energy Detector Spectrum Sensing Algorithms

ROCs curves in Bluetooth sensing with time record length of 50 samples for constant weights (upper) and optimized weights (lower). Analytic and FFT models in AWGN case with -5 dB SNR.

- In these simulation models, the time record length is chosen as 50 samples, due to the hopping limit corresponding to approximately 625 μs.
- The simulated performance is slightly worse than the analytical model.
- To compensate the difference, the time record length should be increased by about 10%.
- But we are still able to reach false alarm and detection probabilities of 0.1 and 0.9 at -5 dB SNR with 50 samples.
Simulation Results of FFT and AFB Based Energy Detector Spectrum Sensing Algorithms

Actual false alarm probability for BT signals with target SNR of -5 dB and $P_{FA}=0.1$ using optimum weight values under AWGN case. (a) BT frequency closest to WLAN. (b) BT frequency at the center between channels 3 and 7. (c) BT frequency at the center of 3 MHz gap.

- The constant weight case uses 3 subbands, which provides the best detection probability performance.

- With optimum weight values using 11 subbands, highest detection probability performance is achieved again, but the benefit over the constant weight case is marginal.

In this low dynamic range case, there is no big difference in detection probability between FFT and AFB.
Simulation Results of FFT and AFB Based Energy Detector Spectrum Sensing Algorithms

Detection probability of Bluetooth signal with target $P_{FA} = 0.1$ using time record length of 50 for constant and optimum weight values in AWGN.

- First a BT channel at the nominal band edge, which includes strong side lobes of the WLAN signal, is investigate in figure 6(a).
- In figure 6(b), the BT in the center frequency between two WLAN signals at channels 3 and 7 is considered to see the effect of spectral leakage.
- Figure 6(c) shows the corresponding case for 3 MHz gap between WLANs at channels 3 and 8.
CONCLUSION

- We have analyzed the performance of energy detection based spectrum sensing techniques with weight process utilizing either FFT or filter bank based spectrum analysis methods.

- Moving average processing is an efficient way to align the sensing time interval with the transmission burst.

- Use of weighting process provides somewhat better performance compared to constant weights.

- If weighting is not used, it is very important to select the number of subbands optimally.

- While there is no big difference between FFT and AFB based spectrum sensing with small spectral dynamic range, the AFB based algorithm has clearly better performance in identifying spectral holes between spectrally well-contained PU signals.

- In the neighborhood of 802.11 WLAN channels, energy detection based spectrum sensing performance depends greatly on the level of spectral regrowth due to WLAN power amplifier nonlinearity.
Part 3 A: FFT and Filter Bank Based Spectrum Sensing and Spectrum Utilization for Cognitive Radio
Block diagram of energy detector with AFB and FFT based spectrum analysis

- In the following analysis, it is assumed that the subband sampling rate is equal to the ADC sampling rate divided by the number of FFT/AFB frequency bins.
AFB & FFT Based Spectrum Sensing Algorithms

\[ Y(m, k) = \begin{cases} W(m, k) & H_0 \\ S(m, k) H_k + W(m, k) H_1 & \end{cases} \]

\[ \tilde{Y}(m, k) = \frac{1}{L_i L_f} \sum_{l=k-[L_f/2]}^{k+[L_f/2]-1} \sum_{u=m-L_f+1}^{m} \left| Y(u, l) \right|^2 \]

\[ f(\tilde{Y}_k)_{|_{H_0}} \sim N\left(\sigma_n^2, \frac{1}{L_i L_f}\sigma_n^4\right) \]

\[ f(\tilde{Y}_k)_{|_{H_1}} \sim N\left(\sigma_k^2 + \sigma_n^2, \frac{1}{L_i L_f}\left(\sigma_k^2 + \sigma_n^2\right)^2\right) \]

\[ \lambda = Q^{-1}(P_{FA}) \sqrt{\frac{\sigma_n^4}{L_i L_f}} + \sigma_n^2 \]

\[ P_{FA} = \Pr(\tilde{Y}_k > \lambda \mid H_0) = Q\left(\frac{\lambda - \sigma_n^2}{\sqrt{\sigma_n^4 / L_i L_f}}\right) \]

\[ P_D = \Pr(\tilde{Y}_k > \lambda \mid H_1) = Q\left(\frac{\lambda - (\sigma_k^2 + \sigma_n^2)}{\sqrt{(\sigma_n^2 + \sigma_k^2)^2 / L_i L_f}}\right) \]
SPECTRUM UTILIZATION

Block diagram of spectrum utilization with water filling algorithm after spectrum analysis

- Here the rate adaptive loading algorithm is considered as it provides better control of the interference from a CR to the PU receivers.
SPECTRUM UTILIZATION

\[
b = \frac{1}{2} \sum_{n=1}^{N} \frac{e_n g_n}{G} + e_n g_n
\]

\[
b_n = \log_2 \left( 1 + e_n g_n \right) / 2 \quad \text{if} \quad n = 1, 2, \ldots, N^*
\]

\[
K = \frac{1}{N - \hat{e}} \sum_{i=1}^{N} e_n + G \sum_{n=1}^{N} e_n
\]

\[
e_i = K - \frac{G}{g_i} \quad \text{satisfy} \quad e_i \geq 0
\]

\[
e_n = K - \frac{G}{g_n}
\]

\[
\max_{e_n} b = \frac{1}{2} \sum_{n=1}^{N} \frac{e_n g_n}{G} + e_n g_n
\]

subject to: \( \hat{N} e_n^x = \hat{A} e_n^x \)
Two WLAN signal spectra in 2.4 GHz ISM band. (a) WLAN channels 3 and 8, (b) WLAN channels 3 and 7.

- In the first case WLAN1 and WLAN2 signals use channels 3 and 8, respectively, out of the entire 11 different channels.
- For the second case, WLAN1 and WLAN2 occupy channels 3 and 7, respectively.
- The channels don't overlap and there are 8 MHz and 3 MHz spectrum holes available in the two cases, respectively.
- According to the 802.11g based WLAN signal properties, energy leakage can still be expected to these bands, since the allowed relative spectrum densities are 20 ... 25 dB below the signal passband level.
Simulation Results of Spectrum Sensing Algorithms

Figure 1 shows this effect in terms of the number of subbands determined to be empty depending on the WLAN SNR.

Two WLANs are assumed to have the same power level, normalized to 0 dB and the target false alarm probability is chosen as 0.1.

Similar results are valid for the bandwidth of spectral hole, as seen in figure 2.
The results are given for groups of 5 subbands in the center of the spectral hole and right next to the guardbands. In the latter case, the spectrum leakage of the WLAN signal degrades the sensing performance already with low WLAN SNRs. In the center of the 3 MHz or 8 MHz spectral hole, the leakage is not significant. However, the poor frequency selectivity of plain FFT based processing degrades the performance of FFT based sensing with modest and high SNRs, whereas the AFB has clearly better performance.
Simulation Results of Spectrum Sensing Algorithms

- The utilization of the 3 MHz and 8 MHz white spaces between the active WLANs with the rate adaptive algorithm is shown in figure 7.

- Constant 10 dB SNR is used for the CR. The channel estimation is assumed to be perfect and the subband wise noise + interference power estimates are obtained using time filtering length of 50.
Finally, figure 8 shows the achievable data rate in the spectrum gap between two active WLANs, as determined by the rate adaptive algorithm.

Naturally, the number of bits for each subband, has been rounded to the nearest integer value.
CONCLUSION

- We have analyzed the performance of energy detection based spectrum sensing techniques using either FFT or filter bank based spectrum analysis methods and utilizing spectral holes with water filling algorithms.
- Due to low leakage, the AFB spectrum analysis based algorithm has clearly better performance in identifying spectral holes between spectrally well-contained PUs.
- The same is true regarding the noise + interference power estimation needed for rate adaptive bit loading.
- In the neighborhood of WLAN channels, energy detection based spectrum sensing performance depends greatly on the level of spectral regrowth due to WLAN power amplifier nonlinearity.
- However, analyzing this effect in the spectrum exploitation context is left as a topic for future studies.
Part 3 B: Spectrum Sensing And Spectrum Utilization Model For OFDM and FBMC Based Cognitive Radios
Spectrum sensing is a vital part of short-range CR applications.

CRs may sense the local spectrum through dedicated sensors, or the spectrum sensing function can be closely integrated with the receiver RF and signal processing modules of access points and mobile stations.

After the spectrum is sensed, efficient spectrum utilization becomes important. Multicarrier techniques provide a solution for efficient transmission of data over channel with frequency selective fading.

When the Channel State Information (CSI) is known, the transmit power or the data rate can be adapted according to the CSI.

The adaptation algorithms, the so-called called loading algorithms use commonly the water-filling principle. Water-filling solution can be thought of as the curve of inverted channel signal to noise ratio being filled with energy to a constant line.
SIMULATION RESULTS OF FFT AND AFB BASED ENERGY DETECTOR SPECTRUM SENSING ALGORITHMS

Two WLAN and FBMC signals using 3rd and 8th WLAN channels in 2.4 GHz ISM band.
As we can see, with no or modest spectral regrowth, an FBMC primary would allow a clearly higher number of subchannels to be used by the CR system compared to OFDM-based WLAN. Also AFB finds higher number of empty subbands compared to FFT, in reliable way. With the worst-case regrowth allowed by 802.11g, the differences disappear.
The actual false alarm probabilities versus the active primary systems’ SNR, as seen by the CR receiver, can be seen in figure 5.

This is actually the probability that a group of 5 subchannels in the center of the gap would be detected to be occupied due to spectral leakage.
Finally, the achievable data rate in the spectrum gap between two active primary channels, as determined by the rate adaptive algorithm, are shown in figure.

Perfect channel estimation is assumed and the subband-wise SINR (signal to interference plus noise ratio) estimates are obtained using time filtering length of 50 samples.
CONCLUSION

• We have analyzed the performance of energy detection based spectrum sensing techniques using either FFT or filter bank based spectrum analysis methods for both WLAN and FBMC signal models and utilizing spectral holes with water filling algorithms.

• As a spectrum sensing method, AFB has clear benefits due to much better spectral containment of the sub channels.

• One significant benefit of FBMC as a transmission technique in CR systems is that it can utilize narrow spectral gaps in an effective and flexible way.

• On the other hand, FBMC multicarrier eliminates the extra complexity due to AFB design because of its transmitter and receiver characteristics.

• As a conclusion, use of FBMC model, instead of OFDM based WLAN model provides better performance in terms of the spectral leakage problem.
Part 4 A: Performance Analysis of Eigenvalue Based Spectrum Sensing under Frequency Selective Channels
There are many problems which effect the performance of spectrum sensing in practice.

The first problem is that reliable sensing has to be achieved with very low signal-to-noise ratio (SNR).

Secondly, the multipath fading and shadowing cause power fluctuation of the received signal. Variation and unpredictability of the precise noise level at the sensing device is another critical issue, which is called “noise uncertainty”.

Especially, the performance of the traditional energy detector based spectrum sensing methods significantly decreases under noise uncertainty.
The goal of this paper is to investigate the effects of frequency selective channel, considering also the noise uncertainty effects, using traditional energy detector and eigenvalue based spectrum sensing.

We consider a simplified signal scenario, where only Gaussian signal model is used under INDOOR, SUI-1 and ITU-R Vehicular A multipath delay profiles.

The applications of cooperative sensing approaches are left as topics for future studies.
A) Energy detector based spectrum sensing with noise uncertainty

\[ H_0: y(n) = w(n) \sim N(0, \sigma_w^2) \]

\[ H_1: y(n) = s(n) \otimes c(n) + w(n) \sim N(0, \sigma_x^2 + \sigma_w^2) \]

\[ T_y \mid_{H_0} = N(\sigma_w^2, \frac{1}{N} \sigma_w^4) \]

\[ T_y \mid_{H_1} = N(\sigma_x^2 + \sigma_w^2, \frac{1}{N}(\sigma_x^2 + \sigma_w^2)^2) \]

\[ P_D = P(T_y > \gamma \mid H_1) \]

\[ P_{FA} = P(T_y > \gamma \mid H_0) = Q\left(\frac{\gamma - \sigma_w^2}{\sqrt{\frac{1}{N} \sigma_w^4}}\right) \]

\[ \gamma = Q^{-1}(P_{FA}) \sqrt{\frac{1}{N} \sigma_w^4 + \sigma_w^2} \]

\[ P_{FA} = \max_{\sigma^2 \in [(1/\rho)\sigma_w^2, \rho\sigma_w^2]} Q\left(\frac{\gamma - \sigma^2}{\sqrt{\frac{1}{N} (\sigma^2)^2}}\right) \]

\[ = Q\left(\frac{\gamma - \rho \sigma_w^2}{\sqrt{\frac{1}{N} (\rho \sigma_w^2)^2}}\right) \]

\[ P_D = \min_{\sigma^2 \in [(1/\rho)\sigma_x^2, \rho\sigma_x^2]} Q\left(\frac{\gamma - (\sigma_x^2 + \sigma_w^2)}{\sqrt{\frac{1}{N} (\sigma_x^2 + \sigma_w^2)^2}}\right) \]

\[ = Q\left(\frac{\gamma - (\sigma_x^2 + (1/\rho)\sigma_w^2)}{\sqrt{\frac{1}{N} ((\sigma_x^2 + (1/\rho)\sigma_w^2)^2}}}\right) \]
**Spectrum Sensing Techniques**

**B) Eigenvalue Based Spectrum Sensing under different frequency selective channel cases**

\( H_0 : y(n) = w(n) \sim N(0, \sigma_w^2) \)
\( x(n) \)

\( H_1 : y(n) = s(n) \otimes ch + w(n) \sim N(0, \sigma_s^2 + \sigma_w^2) \)

\( \hat{y} = [y(n) \ y(n-1) \ y(n-2) \ldots y(n-ML+1)]^T \),  
\( \hat{s} = [s(n) \ s(n-1) \ s(n-2) \ldots s(n-ML+1)]^T \),  
\( \hat{w} = [w(n) \ w(n-1) \ w(n-2) \ldots w(n-ML+1)]^T \)

\[
\begin{align*}
R_{yy} &= E(\hat{y}\hat{y}^\dagger) \\
R_{ss} &= E(\hat{s}\hat{s}^\dagger) \\
R_{ww} &= E(\hat{w}\hat{w}^\dagger) \\
R_{yy} &= HR_{ss}H^\dagger + R_{ww}
\end{align*}
\]

Algorithm 1

\[
\begin{align*}
R_{yy}(N) &= \frac{1}{N} \sum_{n=ML-1}^{L-2+N} \hat{y}(n)\hat{y}(n)^\dagger \\
R_{ww}(N) &= \frac{1}{N} \sum_{n=ML-1}^{L-2+N} \hat{w}(n)\hat{w}(n)^\dagger \\
\gamma_1 &= \left( \frac{N + \sqrt{ML}}{N - \sqrt{ML}} \right)^2 \left( 1 + \frac{N + \sqrt{ML}}{(NML)^{1/6}} F^{-1}(1 - P_{FA}) \right) \\
\lambda_{max} / \lambda_{min} > \gamma_1
\end{align*}
\]

Algorithm 2

\[
\begin{align*}
T(N) &= \frac{1}{MN} \sum_{n=0}^{NM-1} |y(n)|^2 \\
\gamma_2 &= \left( \frac{1}{NQ^{-1}(P_{FA}) + 1} \right) \frac{N}{(\sqrt{N} - \sqrt{ML})^2} \\
(T(N) / \lambda_{min}) > \gamma_2
\end{align*}
\]
Examples non-oversampled and 2x-oversampled spectral models under Vehicular A channel and AWGN noise.

- Simple Gaussian signal models which includes both non-oversampled and 2x-oversampled signal are shown as seen figure 1.

- In this figure, the signals are shown for the ITU-R Vehicular A channel case [11].

- In our signal model, the bandwidth is chosen as 20 MHz. The Vehicular A channel model has 6 taps the maximum delay spreads is about 2.5 ms.
SIMULATION RESULTS OF SPECTRUM SENSING ALGORITHMS

- Figures show detection probabilities of traditional energy detector and eigenvalue based spectrum sensing under INDOOR channel.

- 1 dB noise uncertainty is chosen to see the performance of detection probabilities for 10000 numbers of samples.
Simulation Results of Spectrum Sensing Algorithms

Figure shows detection probabilities of traditional energy detector and eigenvalue-based spectrum sensing under Vehicular channel.

- 1 dB noise uncertainty is chosen to see the performance of detection probabilities for 10000 numbers of samples.
Figures show detection probabilities of traditional energy detector and eigenvalue based spectrum sensing under SUI channel.

1 dB noise uncertainty is chosen to see the performance of detection probabilities for 10000 numbers of samples.
Simulation Results of Spectrum Sensing Algorithms

- Figures show detection probabilities of traditional energy detector and eigenvalue based spectrum sensing under SUI channel.

- 1 dB noise uncertainty is chosen to see the performance of detection probabilities for 10000 numbers of samples.
CONCLUSION

- We have analyzed the performance of the traditional energy detector and eigenvalue based spectrum sensing techniques under different frequency selective channels, the Indoor, ITU-R Vehicular A and SUI-1 channel models in particular.

- It was seen that max/min eigenvalue approach gives consistently better detection performance that energy/min eigenvalue approach. Especially, in simulation based results with oversampling the difference is significant.

- We have seen that eigenvalue based spectrum sensing clearly exceeds the performance of energy detector with 1 dB noise uncertainty with Indoor and Vehicular-A channel models, whereas with SUI-1, the difference is rather small.

- Using oversampled signal model in detection clearly reduces the false alarm probability with eigenvalue based sensing.
One related general aspect regarding spectrum sensing is the following: When the sensing station has a line-of-sight (LOS) connection, the channel can be expected to be mildly frequency selective, but also the power level is high due to lower path loss.

When the sensing station does not have a LOS connection, the signal level is lower, but also the channel can be expected to be highly frequency selective.

Thus, in case of shadowing, the PU signal can be detected using the eigenvalue based approach without essential limitations due to noise uncertainty. In case of LOS channel, simple energy detection based approach might be sufficient.
Part 4 B: Reducing Computational Complexity of Eigenvalue Based Spectrum Sensing for Cognitive Radio
Spectrum sensing of primary users under very low signal-to-noise ratio (SNR) and noise uncertainty is crucial for cognitive radio (CR) systems.

To overcome the drawbacks of weak signal and noise uncertainty, eigenvalue-based spectrum sensing methods have been proposed for advanced CRs.

However, one pressing disadvantage of eigenvalue-based spectrum sensing algorithms is their high computational complexity, which is due to the calculation of the covariance matrix and its eigenvalues.

In this study, power, inverse power and fast Cholesky methods for eigenvalue computation are investigated as potential methods for reducing the computational complexity.
TRADITIONAL EIGENVALUE-BASED SPECTRUM SENSING

\[ H_0 : y(n) = w(n) \approx N(0, \sigma_w^2) \]
\[ H_1 : y(n) = s(n) \otimes h(n) + w(n) \approx N(0, \sigma_x^2 + \sigma_w^2) \]

\[ \begin{align*}
R_{yy} &= E(\hat{y}\hat{y}^\dagger); & R_{ss} &= E(\hat{s}\hat{s}^\dagger); & R_{ww} &= E(\hat{w}\hat{w}^\dagger) \\
R_{yy} &= H_c R_{ss} H_c^\dagger + R_{ww}
\end{align*} \]

Algorithm 1: Max-Min eigenvalue based sensing (MME)

\[ R_{yy}(N) = \frac{1}{N} \sum_{n=ML-1}^{L-2+N} \hat{y}(n)\hat{y}(n)^\dagger \]

\[ \gamma_1 = \left( \frac{\sqrt{N} + \sqrt{ML}}{\sqrt{N} - \sqrt{ML}} \right)^2 \left( 1 + \frac{\sqrt{N} + \sqrt{ML}}{(NML)^{1/6}} F_1^{-1}(1 - P_{FA}) \right) \]

When \( \lambda_{\text{max}} \lambda_{\text{min}} > \gamma_1 \), the primary signal is assumed to be present, otherwise it is assumed that there is no transmitted signal in the band of interest at this time.

Algorithm 2: Energy with min eigenvalue based sensing (EME)

\[ T(N) = \frac{1}{MN} \sum_{n=0}^{NM-1} |y(n)|^2 \]

\[ \gamma_2 = \left( \sqrt{\frac{1}{MN}Q^{-1}(P_{FA})+1} \right) \frac{N}{(\sqrt{N} - \sqrt{ML})^2} \]

When \( T(N)/\lambda_{\text{min}} > \gamma_2 \), the signal is assumed to be present, otherwise it is expected that there is only noise in the band of interest.
PROPOSED EIGENVALUE BASED ALGORITHMS

The Power Method to find the largest eigenvalue

The power method is an iterative algorithm which approximates the largest dominant eigenvalue of a symmetric positive definite matrix in $O(kML)$ operations, where $k$ is the number of iterations under a certain error threshold.

**Input:** $R_{yy}$, the matrix; $v_0$, an initial guess of an eigenvector such that $\|v_0\| = 1$

**For** $k = 1, 2, 3, \ldots$, **do**

- $w \leftarrow R_{yy} v_{k-1}$
- $v_k \leftarrow \frac{w}{\|w\|}$
- $\lambda_k \leftarrow v_k^T R_{yy} v_k$

**End for**

**Output:** $\lambda_k$, the approximation of the maximum eigenvalue of $R_{yy}$ after the $k^{th}$ iteration.

The Inverse Power Method to find the smallest eigenvalue

The inverse power method is an iterative algorithm which approximates the smallest eigenvalue, without finding and sorting all eigenvalues.

Theorem 1: Let $R_{yy}$ be a non-singular $m \times m$ matrix i.e. are non-zero for all $1 \leq i \leq m$ then $R_{yy}^{-1}$ has eigenvalues $1/\lambda_i$ for all $1 \leq i \leq m$

**Proof:** $\in R_{yy}$ is non-singular, there exists $\lambda_i \neq 0$ and there exists $x \neq 0$ such that

$$R_{yy} x = \lambda_i x \Rightarrow R_{yy}^{-1}R_{yy} \left(\frac{1}{\lambda_i} x\right) = R_{yy}^{-1}x$$

$$\Rightarrow \frac{1}{\lambda_i} x = R_{yy}^{-1}x$$
The inverse power method is therefore applying the power method on the inverse of the matrix for approximating the smallest dominant eigenvalue.

However, computing $R_{yy}^{-1}$ explicitly is numerically expensive and unstable, and generally takes $O(M^4L^4)$ operations using naive Gaussian elimination.

A common implementation of the inverse power iteration uses LU decomposition, which has a lower complexity of $O(4/3M^3L^3)$.

Hence, the overall computational complexity of the inverse power method is $O(M^3L^3)$, as the complexity of LU factorization dominates.

To reduce the complexity of the calculations, well known traditional iterative Schur algorithm is proposed in this study.

Computational complexity can be decreased as $O(M^2L^2)$ with iterative Schur algorithm.
Difference of computed smallest eigenvalue using Schur or Cholesky based inverse power methods in comparison with eigen-decomposition method, for varying matrix dimension under different SNR cases.

**Simulation Parameters:**

- Number of samples : $N = 10000$
- Smoothing factor : $L = 16$
- The bandwidth : $20 \text{ Mhz}$
- Max delay spread : $2.5 \text{ ms}$
- The number of ite. : $k=100$
Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 1$ (non-oversampled), $L = 16$ and Indoor channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.

Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 4$ (4x-oversampled), $L = 16$ and ITUR-A vehicular channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.
## Computational Complexity of Power Iteration Based Spectrum Sensing

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<td>Alg. 1</td>
<td>MLN</td>
<td>O(M^3L^3)</td>
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<td>MLN + O(M^3L^3)</td>
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<td>(max. eig. / min. eig.)</td>
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<td>Alg. 2</td>
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<td>O(M^3L^3)</td>
<td>MN</td>
<td>MLN + O(M^3L^3) + MN</td>
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<td>(average / min. eig.)</td>
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<tr>
<td>Alg. 1</td>
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<td>O(kML)</td>
<td>O(M^3L^3 / 3)</td>
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<td>Alg. 2</td>
<td>MLN</td>
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<td>Schur Algorithm</td>
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### SIMULATIONS AND NUMERICAL RESULTS

#### SOME NUMERICAL VALUES OF COMPUTATIONAL COMPLEXITIES FOR SENSING METHODS (M=1, NON-OVERSAMPLED)

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<td>Alg. 2 (average / min. eig.)</td>
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<td>Alg. 1 (max. eig. / min. eig.)</td>
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<td>Alg. 2 (average / min. eig.)</td>
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### SOME NUMERICAL VALUES OF COMPUTATIONAL COMPLEXITIES FOR SENSING METHODS (M=2X-OVERSAMPLED)

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<td>40 512</td>
<td>164 096</td>
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<td>16 35 024</td>
<td>85 256</td>
<td>341 024</td>
<td>681 024</td>
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**Alg. 1**

**Alg. 2**
### SOME NUMERICAL VALUES OF COMPUTATIONAL COMPLEXITIES FOR SENSING METHODS (M=4X-OVERSAMPLED)

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<td>Schur</td>
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CONCLUSION

- An improvement to the computational complexity of eigenvalue-based spectrum sensing has been presented in this paper, based on the simple power iteration and inverse power iteration using the Schur algorithm.

- In general, the max/min eigenvalue method provides consistently better performance/complexity tradeoff than the energy with min eigenvalue method.

- On the other hand, the Schur algorithm has in many cases significantly lower complexity than the Cholesky approach.

- When the number of samples is 1000 with 4x-oversampling, the overall computational complexity (multiplication and addition) of the traditional algorithm 1 is 326144 whereas it is 74496 Schur algorithm. Hence upon using the Schur algorithm, the complexity is reduced by about 80 percent.

- Besides numerical algorithms, other aspects of cognitive radio, such as cooperative sensing will also be investigated to reduce the computational complexity as a future work.
Part 4 C: Reduced Complexity Spectrum Sensing Based on Maximum Eigenvalue and Energy
Spectrum sensing of primary users under very low signal-to-noise ratio (SNR) and noise uncertainty is crucial for cognitive radio (CR) systems.

Eigenvalue-based spectrum sensing methods have been proposed for advanced cognitive radios to solve the low SNR and noise uncertainty challenges.

One very crucial drawback of eigenvalue based spectrum sensing techniques is their high computation complexity.

In this paper, power methods for eigenvalue computation are studied with the proposed energy with maximum eigenvalue (EMaxE) detection as potential methods for reduced complexity.
TRADITIONAL EIGENVALUE-BASED SPECTRUM SENSING

\[ H_0 : y(n) = w(n) \approx N(0, \sigma_w^2) \]
\[ H_1 : y(n) = s(n) \otimes h(n) + w(n) \approx N(0, \sigma_x^2 + \sigma_w^2) \]

\[ \mathbf{R}_{yy} = E(\hat{\mathbf{y}}\hat{\mathbf{y}}^\dagger); \quad \mathbf{R}_{ss} = E(\hat{\mathbf{s}}\hat{\mathbf{s}}^\dagger); \quad \mathbf{R}_{ww} = E(\hat{\mathbf{w}}\hat{\mathbf{w}}^\dagger) \]

\[ \mathbf{R}_{yy} = \mathbf{H}_c \mathbf{R}_{ss} \mathbf{H}_c^\dagger + \mathbf{R}_{ww} \]

Algorithm 1: Max-Min eigenvalue based sensing (MME)

\[ \mathbf{R}_{yy}(N) = \frac{1}{N} \sum_{n=ML-1}^{L-2+N} \hat{y}(n)\hat{y}(n)^\dagger \]

\[ \gamma_1 = \frac{(\sqrt{N} + \sqrt{ML})^2}{(\sqrt{N} - \sqrt{ML})^2} \left(1 + \frac{(\sqrt{N} + \sqrt{ML})^{-2/3}}{(NML)^{1/6}} F_1^{-1}(1 - P_{FA}) \right) \]

When \( (\lambda_{\text{max}} / \lambda_{\text{min}}) > \gamma_1 \), the primary signal is assumed to be present, otherwise it is assumed that there is no transmitted signal in the band of interest at this time.

Algorithm 2: Energy with min eigenvalue based sensing (EME)

\[ T(N) = \frac{1}{MN} \sum_{n=0}^{NM-1} |y(n)|^2 \]

\[ \gamma_2 = \left( \frac{1}{\sqrt{MN} Q^{-1}(P_{FA}) + 1} \right) \frac{N}{(\sqrt{N} - \sqrt{ML})^2} \]

When \( (T(N)/\lambda_{\text{min}}) > \gamma_2 \), the signal is assumed to be present, otherwise it is expected that there is only noise in the band of interest.
Energy with max eigenvalue based sensing (EmaxE)

The threshold value \( \gamma \) can be calculated using the random matrix theorem. The probability of false alarm of the EmaxE detection can be expressed as

\[
P_{FA} = P(\lambda_{max} > \gamma T(N))
\]

\[
= P(A(N)) > \gamma T(N) \frac{N}{\sigma_w^2}
\]

\[
= P\left(\frac{\lambda_{max}(A(N)) - \mu}{\nu} > \frac{\gamma T(N)N/\sigma_w^2 - \mu}{\nu}\right)
\]

\[
\gamma = \frac{\sqrt{N+\sqrt{ML}}}{{F^{-1}(1-P_{FA})}} \left(\frac{\sqrt{ML+\sqrt{N}}}{\sqrt{N+\sqrt{ML}}}\right)^{\frac{1}{3}} (\sqrt{N+\sqrt{ML}})
\]

When \( (\lambda_{max}/T(N)) > \gamma \), the signal is assumed to be present, otherwise it is expected that there is only noise in the band of interest. This algorithm is very robust against noise uncertainty because it doesn’t need knowledge about noise variance.

The Power Method to find the largest eigenvalue

The power method is an iterative algorithm which approximates the largest dominant eigenvalue of a symmetric positive definite matrix in \( O(kML) \) operations, where \( k \) is the number of iterations under a certain error threshold.
The detection probabilities of simple energy detector, as well as traditional and proposed eigenvalue based spectrum sensing methods have been evaluated using three different channel models (i.e., Indoor, ITU-R Vehicular A and SUI-1 channels). The dB noise uncertainty case is considered as the worst-case scenario in terms of noise variance estimation. The Vehicular A channel has 6 taps and its maximum delay spreads is about 2.5μs. 16-tap model with 80 ns rms delay spread is also applied for realistic Indoor channel model as second channel model. SUI-1 channel model which has 3 Ricean fading taps and 0.9 delay spread is used as third channel model.

Some Simulation Parameters:

- Number of samples: \( N = 1000 \)
- Smoothing factor: \( L = 16 \)
- The bandwidth: \( 20 \ \text{MHz} \)
- Max delay spread: \( 2.5 \ \text{ms} \)
- The number of ite.: \( k = 100 \)
Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 1$ (non-oversampled), $L = 16$ and Indoor channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.

Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 4$ (4x-oversampled), $L = 16$ and Indoor channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.
Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 1$ (non-oversampled), $L = 16$ and ITUR-A vehicular channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.

Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 4$ (4x-oversampled), $L = 16$ and ITUR-A vehicular channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.
Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 1$ (non-oversampled), $L = 16$ and SUI-1 channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.

Simulated detection probabilities using traditional and proposed eigenvalue-based spectrum sensing algorithms with $M = 4$ (4x-oversampled), $L = 16$ and SUI-1 channel. Theoretical performance of energy detector without noise uncertainty and with 1 dB noise uncertainty included as reference.
Actual false alarm probability of traditional and proposed eigenvalue based spectrum sensing algorithms with $M = 1$ (non-oversampled) and oversampling by $M = 4$, $L = 16$ and Indoor channel.

False alarm probability is not changing with SNR as expected. While the actual false alarm probabilities under the oversampled signal models are very small, the detection probabilities of proposed eigenvalue based spectrum sensing is clearly better compared to the traditional eigenvalue based methods.
### Computational Complexity of Power Iteration Based Spectrum Sensing

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Trad. Alg.:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alg. 1 (max. eig. / min. eig.)</td>
<td>MLN</td>
<td>$O(M^3 L^3)$</td>
<td></td>
<td>$MLN + O(M^3 L^3)$</td>
</tr>
<tr>
<td>Alg. 2 (average / min. eig.)</td>
<td>MLN</td>
<td>$O(M^3 L^3)$</td>
<td>MN</td>
<td>$MLN + O(M^3 L^3) + MN$</td>
</tr>
<tr>
<td><strong>Prop. Alg.:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EmaxE (max. eig. / average)</td>
<td>MLN</td>
<td>$O(kML)$</td>
<td>MN</td>
<td>$MLN + O(kML) + MN$</td>
</tr>
</tbody>
</table>

**TABLE** shows expressions for the computational complexities of traditional methods (based on finding all the eigenvalues) and the proposed eigenvalue based spectrum sensing technique using the max eigenvalue over energy criterion together with power iteration. Here the metric for computational complexity is the overall number of multiplications and additions.
### SOME NUMERICAL VALUES OF COMPUTATIONAL COMPLEXITIES FOR SENSING METHODS (M=1, NON-OVERSAMPLED)

<table>
<thead>
<tr>
<th>ALGORITHMS</th>
<th>Smoothing Factor (L)</th>
<th>Number of samples (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10^3</td>
</tr>
<tr>
<td>Trad. Alg.</td>
<td>Alg. 1 (max. eig./min. eig.)</td>
<td>8</td>
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<tr>
<td></td>
<td></td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Alg. 2 (energy/min. eig.)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>Prop. Alg.</td>
<td>EmaxE (max. eig./energy)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
</tr>
</tbody>
</table>

### SOME NUMERICAL VALUES OF COMPUTATIONAL COMPLEXITIES FOR SENSING METHODS (M=4, 4x-OVERSAMPLED)

<table>
<thead>
<tr>
<th>ALGORITHMS</th>
<th>Smoothing Factor (L)</th>
<th>Number of samples (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>
The EMaxE eigenvalue based spectrum sensing technique with power iteration has been presented in this paper.

We have analyzed and simulated both false alarm and detection probability performances of the traditional and proposed EmaxE eigenvalue based spectrum sensing methods under different frequency selective channel models.

The overall computational complexity can be reduced from $O((ML)^3)$ to $O(kML)$ using the EMaxE algorithm.

When the number of samples is 1000 with $L = 16$ and $M = 4$, the overall computational complexity (number of multiplications and additions) of the traditional max/min algorithm is 326144 whereas it is 74400 for the EmaxE algorithm.

Besides Hence upon using the EmaxE algorithm, the complexity is reduced by about 77 percent. Also the sensing performance is clearly improved.
THANK YOU VERY MUCH FOR YOUR LISTENING …

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Tampere University of Technology