Carrier Phase Tracking at Low Signal-to-Noise Ratio And A Performance Comparison of a Parity-Code-Aided and a Pilot-Symbol-Assisted Approach

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Carrier synchronization at very low SNR

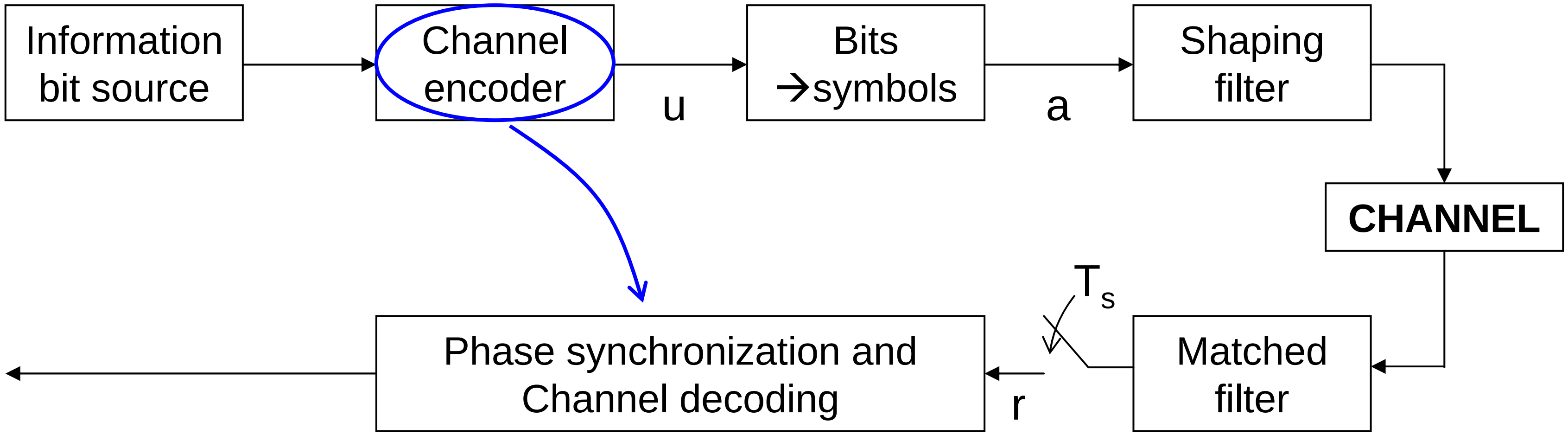
- Very powerful channel codes
  - → Communications at very low SNR
- Correct decoding requires sufficient synchronization performance
  - → Conventional synchronizers are not accurate enough
  - → Carrier synchronization must be improved

→ New proposed synchronization schemes are based on the exploitation of some form of *a priori* information

Problem formulation (2)

Carrier synchronization at very low SNR

- Approach 1:
  Use the channel code properties for synchronization



Example: Turbo-synchronization

Problem formulation (3)

Carrier synchronization at very low SNR

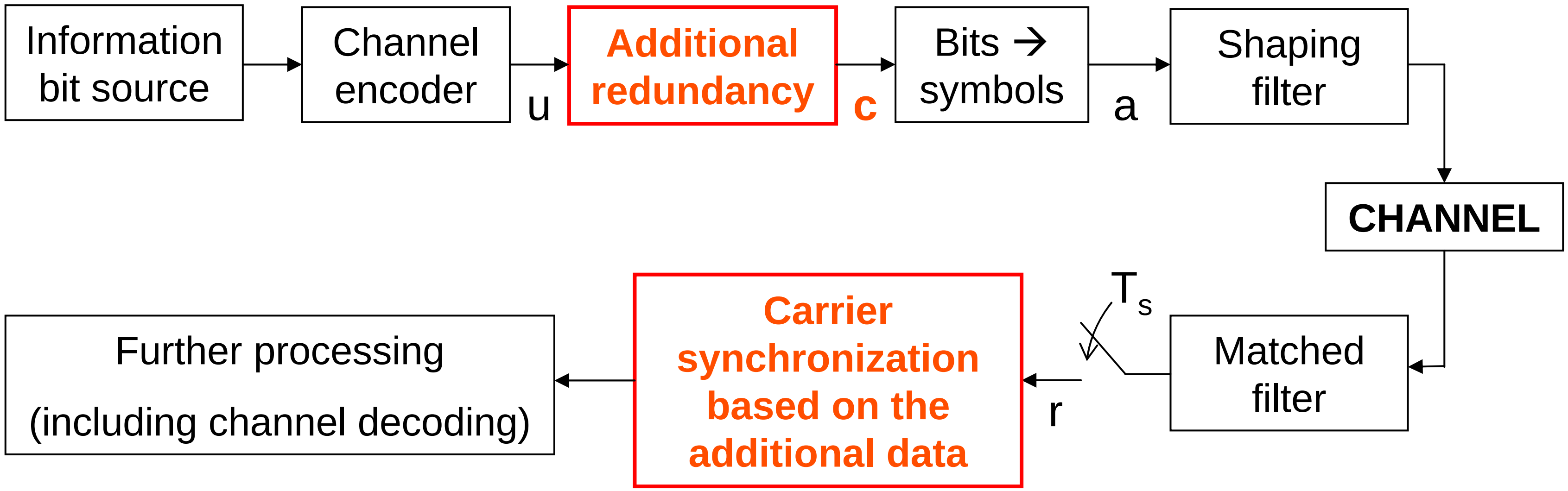
- Approach 1:
  Use the channel code properties for synchronization

Phase recovery without a priori information is necessary prior to the first decoding operation

→ This is problematic with long coded sequences and a time-varying phase error

Carrier synchronization at very low SNR

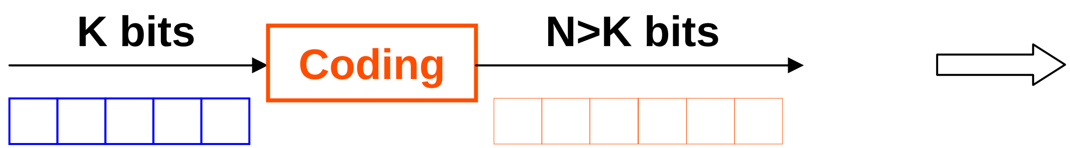
- Approach 2:
  Introduce additional redundancy for the sake of synchronization



Carrier synchronization at very low SNR

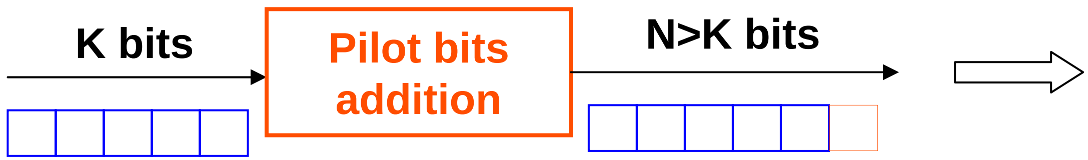
- Approach 2:
  Introduce additional redundancy for the sake of synchronization:

➢ Redundancy brought by additional channel coding with rate R=K/N:



Correlation is introduced into the data

➢ Redundancy brought by deterministic pilot bits independent of the user data:



No correlation between the pilot bits and the user bits

Problem formulation (6)

Objectives:

- Compare the performance of feedback synchronizers based on additional redundancy brought by:
  - A single parity-check (SPC) code
  - Deterministic pilot bits
- The same reduction of spectral and power efficiency is considered in both schemes.
- Synchronization performances are compared:
  - for a constant phase error
  - in presence of phase noise
- A Gaussian channel model is considered

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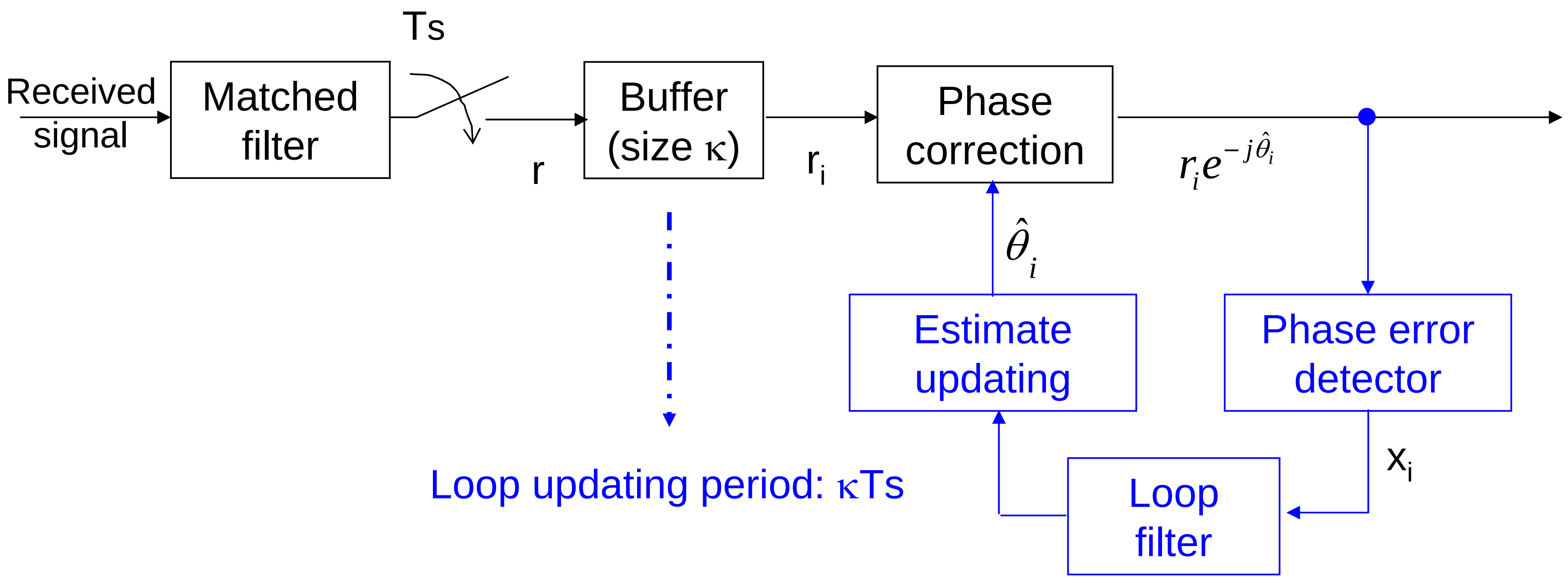
Phase locked loop structure



Phase detector output

- The phase loop aims at maximizing the phase likelihood
  → by setting the derivative of the likelihood function to zero

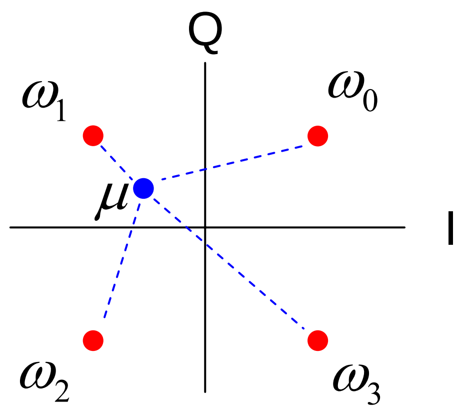
- General expression of the phase detector output:

$$x_i \propto \frac{1}{\kappa} \sum_{l=i\kappa}^{(i+1)\kappa-1} \mathrm{Im}\left(\mu^*\left(l; r_i, \hat{\theta}_i\right) r(l) e^{-j\hat{\theta}_i}\right)$$

where

$$\mu\left(l; r_i, \hat{\theta}_i\right) = \sum_{m=0}^{M-1} \underbrace{\Pr\left(a(l) = \omega_m \middle| r_i, \hat{\theta}_i\right)}_{\text{APP}} \omega_m$$

$r_i e^{-j\tilde{\theta}_i}$: κ samples at the detector input



(« soft » decision)

1 - PLL with Non-Code-Aided (NCA) operation

(Independent) received symbols (=> κ=1): r(0) | r(1) | r(2) ...

Computation of the APPs:

$$\Pr\left(a(0)=\omega_m \middle| r_i,\hat{\theta}_i\right) \propto \exp\left(-\frac{1}{N_0}\left|r(0)-\omega_m e^{j\hat{\theta}_i}\right|^2\right)$$

transmitted symbol ($a(0)$)

Constellation symbol (ω_m)

$F(r(0),\omega_m e^{j\theta})$

« Soft » Decisions: $$\mu\left(0;r_i,\hat{\theta}_i\right)=\sum_{m=0}^{M-1}\Pr\left(a(0)=\omega_m \middle| r_i,\hat{\theta}_i\right)\omega_m$$

Detector output: $$x_i \propto \mathrm{Im}\left(\mu^*\left(0;r_i,\hat{\theta}_i\right)r(0)e^{-j\hat{\theta}_i}\right)$$

2 - PLL with Single-Parity-Check Code-Aided (SPC-CA) operation

(Correlated) received symbols: r(0) | r(1) | r(2) | ▪ ▪ ▪ | r(κ-1)

Computation of the APPs:

$$\Pr\left(a(0)=\omega_m \middle| r_i,\hat{\theta}_i\right) \propto \underbrace{F\left(r(0),\omega_m e^{j\hat{\theta}_i}\right)}_{\text{Channel probability}} \underbrace{G\left(r(1),\dots,r(\kappa-1)\right)}_{\text{Extrinsic probability}}$$

« Soft » Decisions:

$$\mu\left(0;r_i,\hat{\theta}_i\right)=\sum_{m=0}^{M-1}\Pr\left(a(0)=\omega_m \middle| r_i,\hat{\theta}_i\right)\omega_m$$

Detector output:

$$x_0 \propto \operatorname{Im}\left(\mu^*\left(0;r_i,\hat{\theta}_i\right)r(0)e^{-j\hat{\theta}_i}\right)$$

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Example : κ=2, 8PSK, parity code R=5/6

$$\delta_0 = \mu\left(0; r_i, \hat{\theta}_i\right) = \sum_{m=0}^{7} \Pr\left(a(0) = \omega_m \middle| y_0, y_1\right)\omega_m = \sum_{m=0}^{7} P_{0m}^{app}\,\omega_m$$

$$P_{0m}^{app} \propto \Pr\left(a(0) = \omega_m, y_0, y_1\right) = \sum_{j=0}^{7} \Pr\left(a(0) = \omega_m, a(1) = \omega_j, y_0, y_1\right)$$

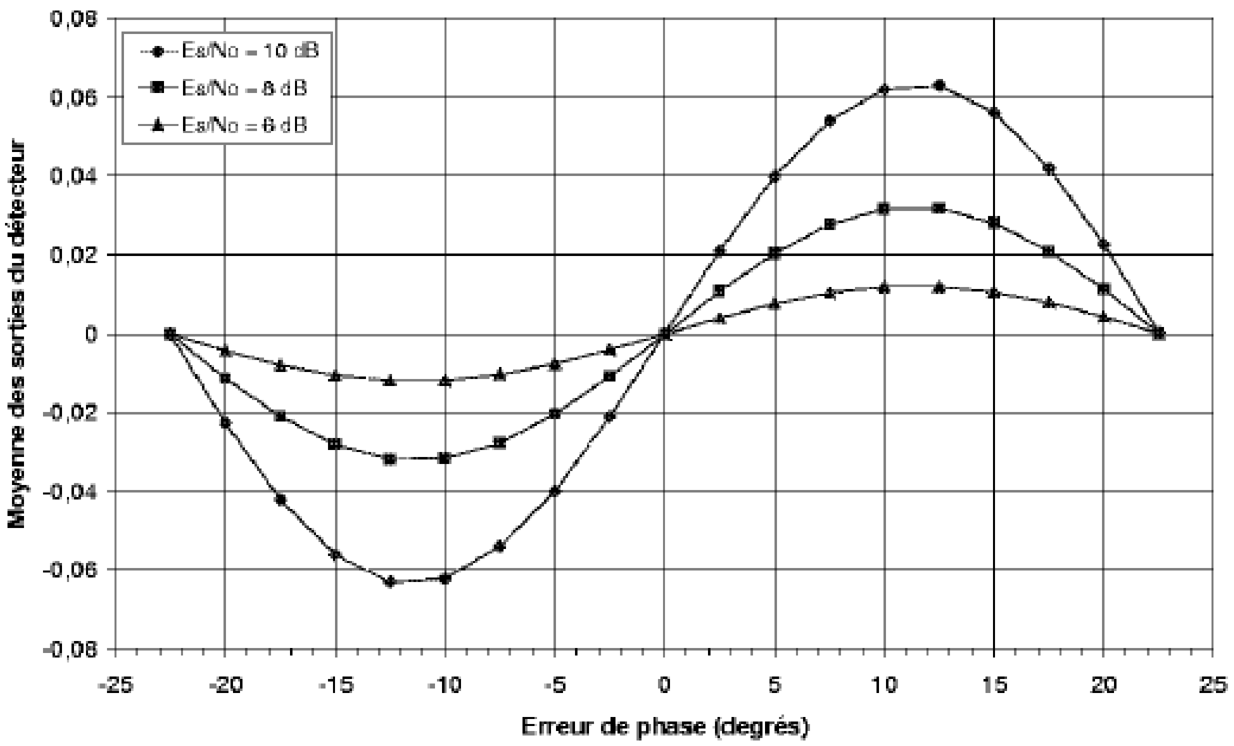
$$\Pr\left(a(0) = \omega_m, a(1) = \omega_j, y_0, y_1\right) = \Pr\left(y_0 \middle| a(0) = \omega_m\right)\Pr\left(y_1 \middle| a(1) = \omega_j\right)\Pr\left(a(0) = \omega_m, a(1) = \omega_j\right)$$

$$\Pr\left(a(0) = \omega_m, a(1) = \omega_j\right) = \Pr(u_0, u_1, \ldots, u_5) = \Pr\left(u_5 \middle| u_0, u_1, \ldots, u_4\right)\prod_{i=0}^{4} P\left(u_i\right) = 0 \text{ si } u_1 \oplus u_2 \oplus \ldots \oplus u_5 \neq 0$$

$$P_{0m}^{app} \propto \underbrace{\Pr\left(y_0 \middle| a(0) = \omega_m\right)}_{\text{Channel probability}} \underbrace{\sum_{j=0}^{3} \Pr\left(y_1 \middle| a(1) = \omega_j^* \right)}_{\text{Extrinsic probability}}$$

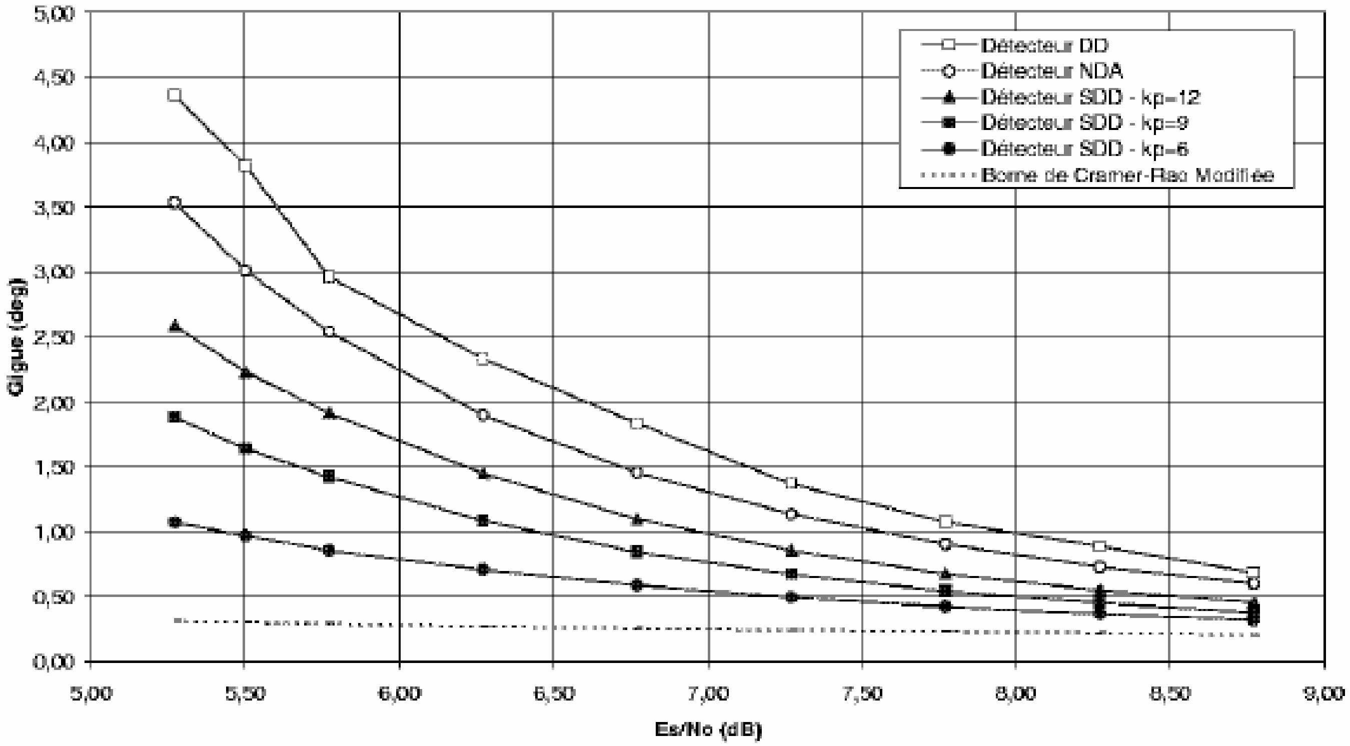
Channel probability

Extrinsic probability



S curve , 8PSK, P=2 , R=5/6

SPC-CA PLL (4)



8PSK, $B_L T_s = 10^{-4}$

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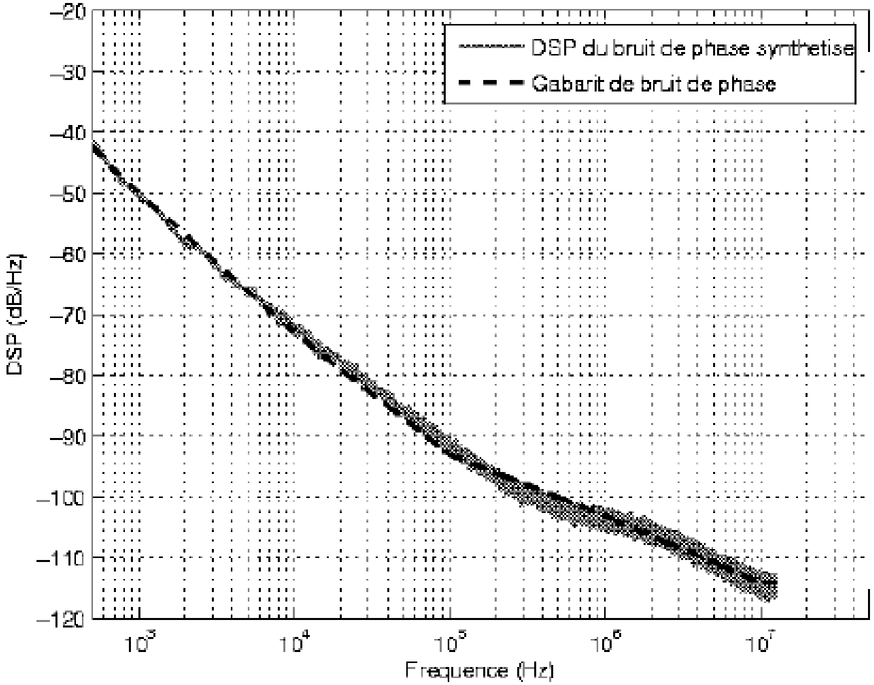
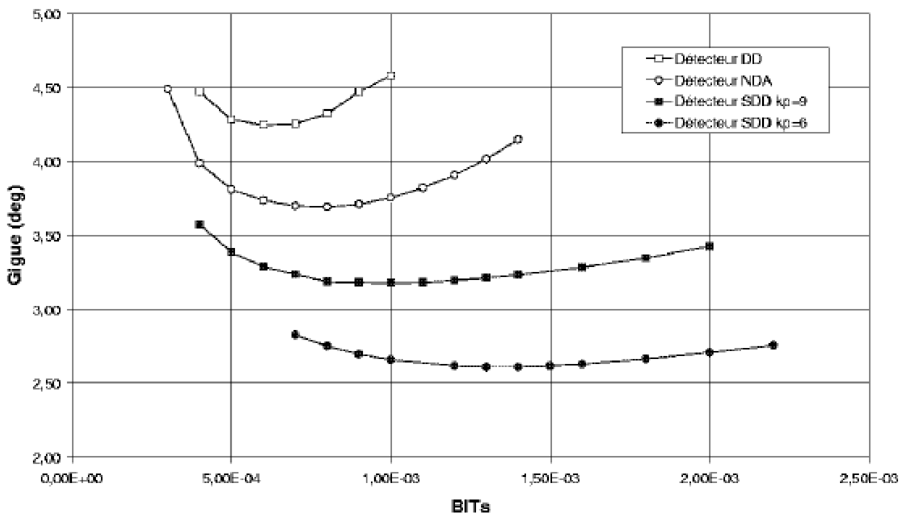
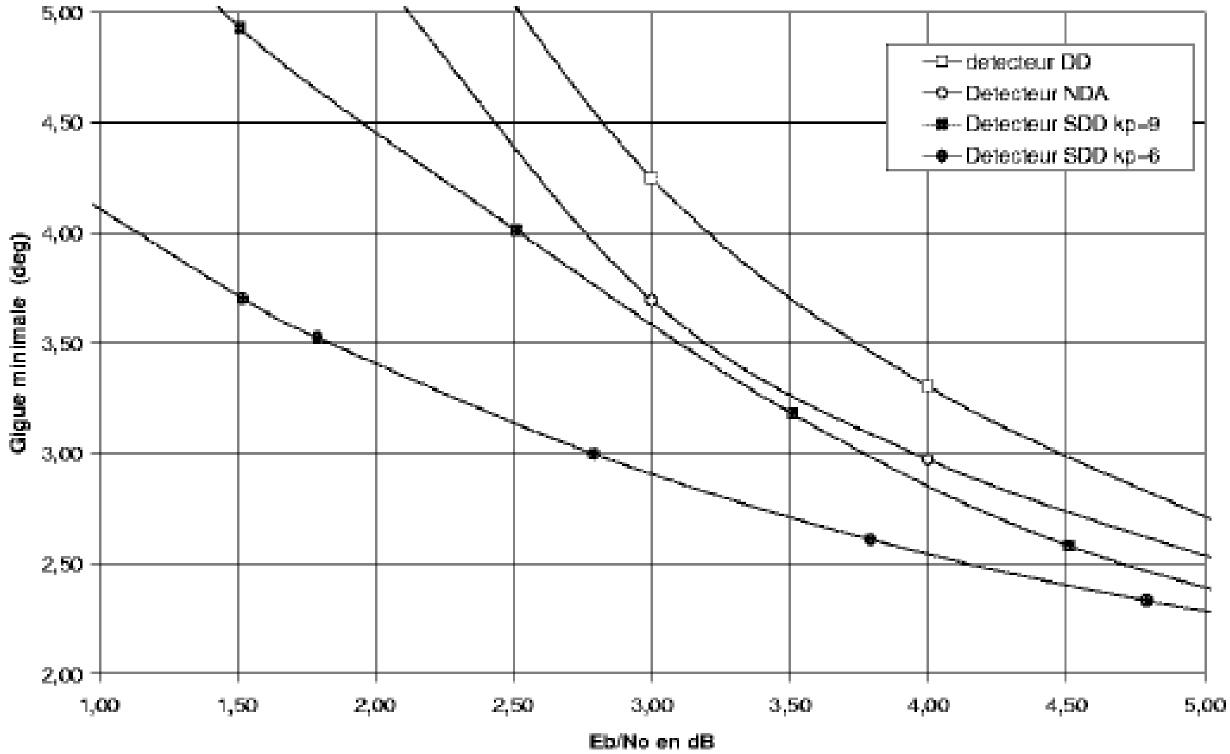




Performance in presence of phase noise, 8PSK, Es/No=7.8 dB



Minimal jitter in function of Eb/No (8PSK)

3 - PLL with Pilot-Bit-Aided (PBA) operation

(Non correlated) received symbols:

r(0)	r(1)	▪ ▪ ▪	r (κ-2)	r(κ-1)

Computation of the APPs:

- L=0,…, κ-2:

$$\Pr\left(a(l)=\omega_m \middle| r_i,\hat{\theta}_i\right) \propto F\left(r(l),\omega_m e^{j\hat{\theta}_i}\right)$$

- L=κ-1:

$$\Pr\left(a(0)=\omega_m \middle| r_i,\hat{\theta}_i\right) \propto \begin{cases} F\left(r(0),\omega_m e^{j\hat{\theta}_i}\right) & \text{for } \omega_m \text{ involving the pilot bit} \\ 0 & \text{otherwise} \end{cases}$$

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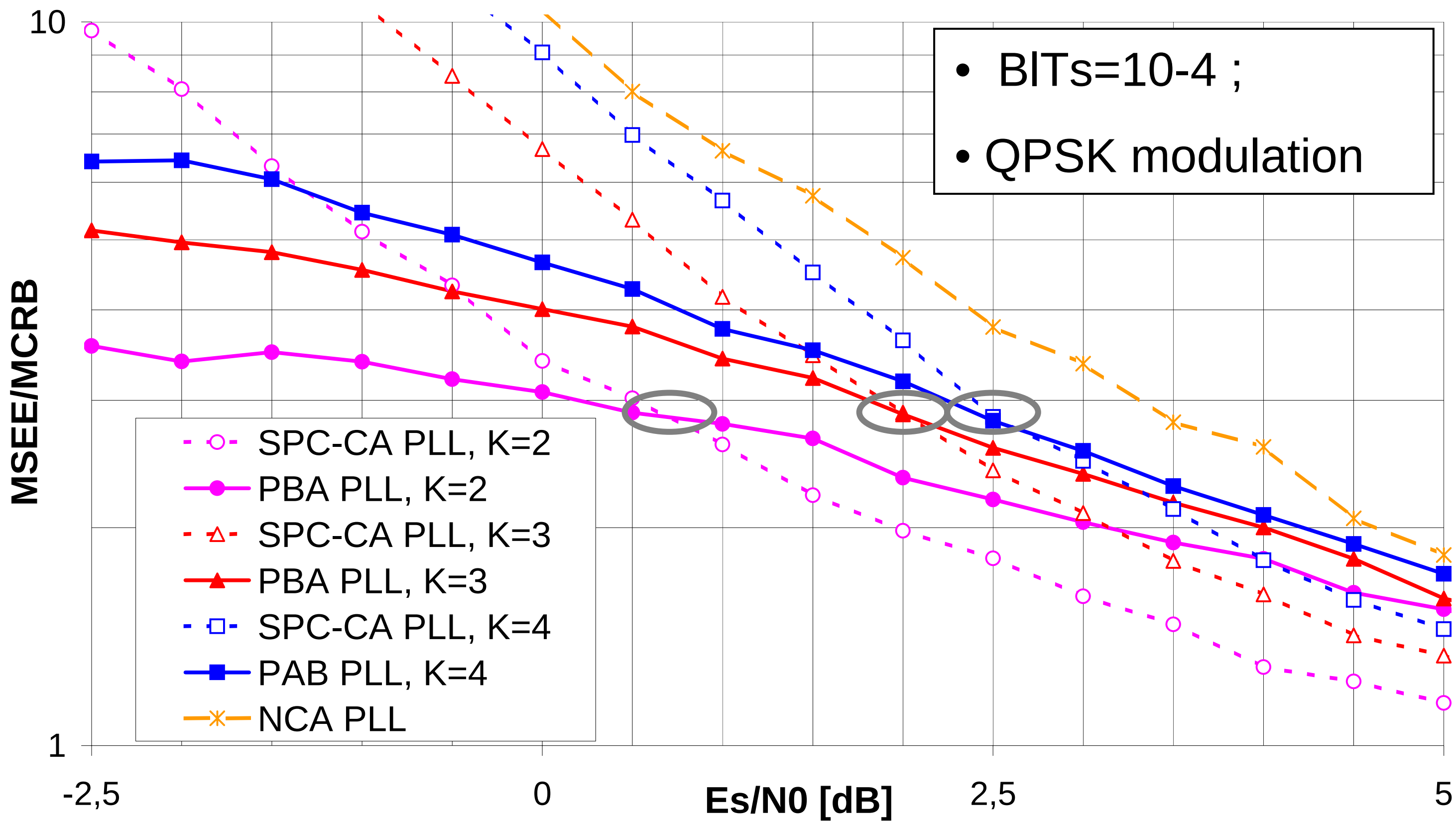
- Proposed feedback synchronizers
  - Phase locked loop (PLL) structure
  - PLL with Non-Code-Aided (NCA) Operation
  - PLL with Single-Parity-Check Code-Aided (SPC-CA) Operation
  - PLL with Pilot-Bit-Aided (PBA) Operation
- Numerical Results for QPSK
  - Estimation of a constant carrier phase
  - Estimation of a time-varying carrier phase

Numerical results (1)

Constant carrier phase



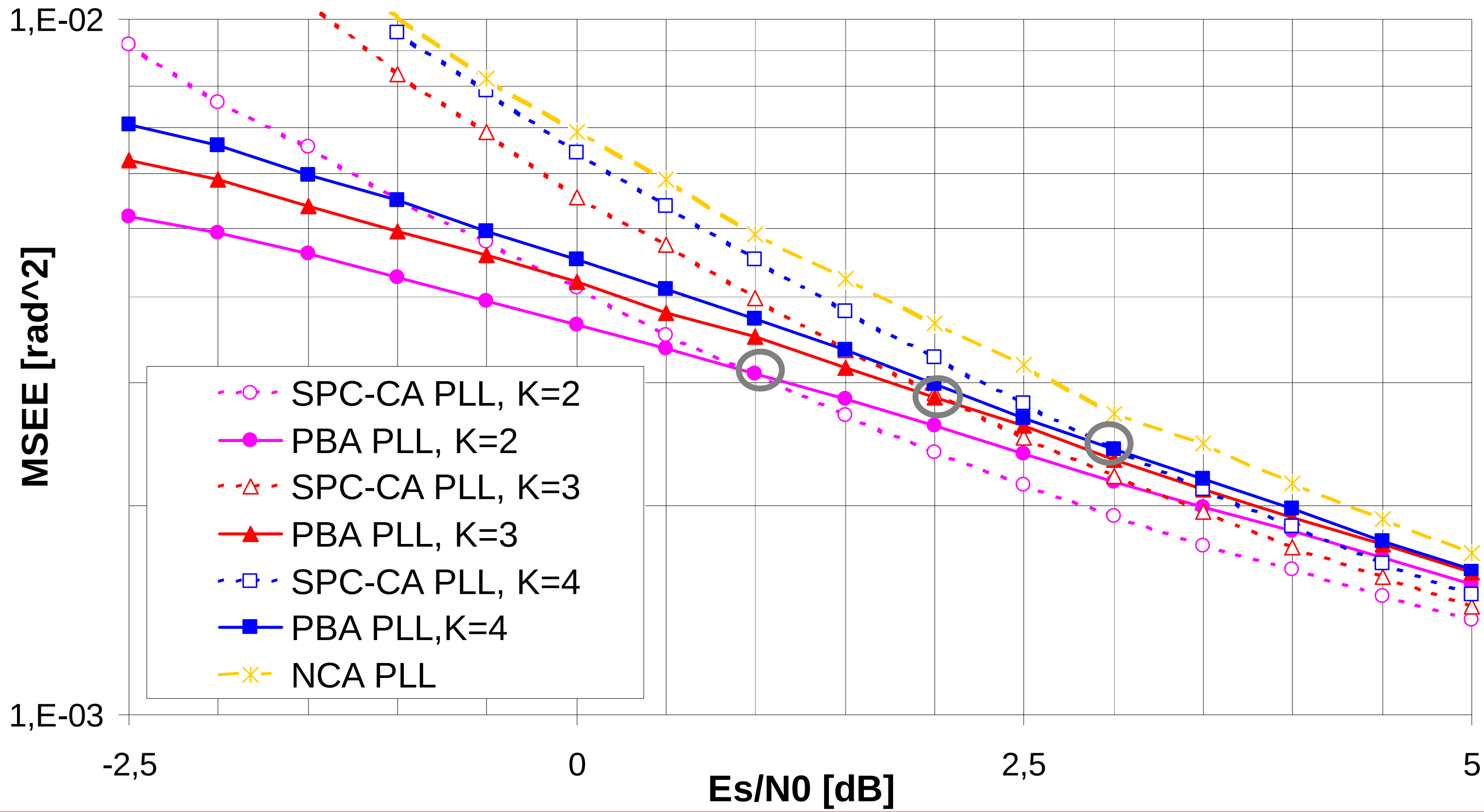
Numerical results (2)

Time-varying carrier phase

- QPSK modulation (gray mapping)
- Phase noise model satisfying the spectral template defined in DVB-S2 standard
- Second order PLL with a damping factor of 0.707
- An optimal loop bandwidth allows to minimize the mean square estimation error (MMSE) in presence of phase noise

Numerical results (3)

Time-varying carrier phase



Numerical results (4)

Conclusions

- Both algorithms exploiting data overhead outperform the non-code aided scheme

- For a given overhead ratio:
  - The SPC-code-aided PLL performs better at moderate to high SNR
  - The Pilot-bit-aided PLL performs better at low SNR

- For a given SNR, the synchronizer performance of both systems improves with decreasing spectral efficiency.

Numerical results (5)

Concluding example

Operation SNRs proposed in DVB-S2 standard:

signaling constellation and code rate	E_s/N_0 required for a PER < 10^{-7}
4-PSK, R=1/4	-2.35 dB
4-PSK, R=1/3	-1.24 dB
4-PSK, R=2/5	-0.30 dB
4-PSK, R=1/2	1.00 dB
4-PSK, R=3/5	2.23 dB
4-PSK, R=2/3	3.10 dB
4-PSK, R=3/4	4.03 dB
4-PSK, R=4/5	4.68 dB
4-PSK, R=5/6	5.18 dB
4-PSK, R=9/10	6.42 dB

for R=3/4 regarding the data overhead:

Pilot-bit-aided PLL is better

SPC-code-aided PLL Is better