Lattice-Reduction-Aided Sphere-Detector as a Solution for Near-Optimal MIMO Detection in Spatial Multiplexing Systems

Sébastien Aubert
ST-ERICSSON Sophia & INSA IETR Rennes
sebastien.aubert@stericsson.com

Supelec Rennes
Signal, Communication et Électronique Embarquée (SCEE)
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1) System introduction and problem statement
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1) System introduction

- Multiple-Input Multiple-Output (MIMO) Spatial Multiplexing case
- System introduction (Narrowband model)

Notations and assumptions:
- Transmit symbol vector $\mathbf{x}$, each symbol is mapped onto a constellation, complex vector of size $n_T$
- Memoryless $\mathbf{H}$ known at receiver, i.i.d. $n_R \times n_T$ complex matrix
- Receive symbol vector $\mathbf{y}$, complex vector of size $n_R$
- Additive White Gaussian Noise (AWGN) $\mathbf{n}$ components are i.i.d.

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$
1) Problem statement (1)

- MIMO Detection step is either
  - The dominant source of complexity, or
  - The dominant source of performance loss, or ...
  - BOTH!

- Joint detection (Maximum Likelihood (ML)):
  \[ x_{ML} = \arg\min_{x} \| y - Hx \|^2 \]
  for all \( x \) in set of possibly transmit symbols vectors
  
  + Optimal performance
  - Exponential complexity \( (M^{nT}) \)
1) Problem statement (2)

- **Linear-Equalization**
  - **ZF:** \( G = (H^H H)^{-1} H^H \) \( (=H^\dagger) \) => \( Gy = G(Hx+n) = x + Gn \)
  - **MMSE:** \( G = (H^H H + 1/\text{SNR} I)^{-1} H^H \)

  + Polynomial complexity
  - noise amplification for ZF, no diversity in reception

- **Successive-Interference Canceller (SIC)**
  - **QRD-based:** \( H = QR \), with \( Q^H Q = I \) and \( R \) is upper triangular
  \n  \[ x_{SIC} = \arg\min \| Q^H y - Rx \|_2^2 \]

  + Polynomial complexity
  - Error propagation

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2) Sphere Decoder (1)

- General principle of Sphere Decoder [AEVZ02]
- Neighborhood study, inside a radius $d$
- QRD-based: $x_{SD} = \text{argmin} \ |Q^H y - Rx| < d^2$
- Unconstrained ZF solution centered: $y_{ZF}$ [WTCM02]

\[
|y - Hx|^2 = |HH^T y - Hx|^2 = |H(y_{ZF} - x)|^2 = |Re|^2
\]

Layer by layer Partial Euclidean Distance (PED) minimization
2) Sphere Decoder (2)

Layer by layer Partial Euclidean Distance (PED) minimization

\[ | |Re| |^2 = \sum_{i=n_T, ..., 1} PED_i \]

\[ PED_i = R'_{i,i} |x_i - y_{ZF,i} + \sum_{j=i+1, ..., n_T} R'_{i,j}(x_{est,i} - y_{ZF,i})|^2 \]

\[ = R'_{i,i} |x_i - z_i|^2 \]

Cumulated Euclidean Distance (CED)

\[ CED_i = PED_i + CED_{i-1} \]

Try \( x_i \) at each layer \( z_i \) is a constant.

+ Implementation interest for PED computation
2) Sphere Decoder (2)

Layer by layer Partial Euclidean Distance (PED) minimization

\[ | | R e | |^2 = \sum_{i=n_T, \ldots, 1} | | R_{i,i} e_i | |^2 = \sum_{i=n_T, \ldots, 1} | | R_{i,i} e_i + \sum_{j=i+1, \ldots, n_T} R_{i,j} e_j | |^2 \]

\[ = \sum_{i=n_T, \ldots, 1} | | R_{i,i} | |^2 | | e_i + \sum_{j=i+1, \ldots, n_T} R_{i,j}/R_{i,i} e_j | |^2 \]

\[ = \sum_{i=n_T, \ldots, 1} \left( R'_{i,i} | | e_i + \sum_{j=i+1, \ldots, n_T} R'_{i,j} e_j | |^2 \right) \]

\[ = \sum_{i=n_T, \ldots, 1} PED_i \]

\[ PED_i = R'_{i,i} | x_i - z_i |^2 \]

Cumulated Euclidean Distance (CED)

\[ CED_i = PED_i + CED_{i-1} \]

Try \( x_i \) at each layer \( z_i \) is a constant.

+ Implementation interest for PED computation
2) Sphere Decoder (3)

- Constant radius, Fincke-Pohst enumeration

- Arbitrary constellation exploration [HV05]

  + Complexity limitation of ML algorithm, Optimal detector
  - Problem of radius choice on performance

- Shrank radius, Schnorr-Euchner enumeration

  + Reduced complexity, independent of radius $d$, Optimal detector
  - Problem of variable complexity, complexity depends on SNR and channel conditions, and depth-first search

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2) Sphere Decoder (4)

- **K-Best Sphere Decoder**
  - The $K$ candidates with the smallest Euclidean distance are stored

  $x_{nT}$

  $x_{nT-1}$

  $x_{nT-2}$

  + **Fixed Complexity, parallel algorithm**
  - **$K$ value for high order constellations (16QAM, 64QAM), non-optimal detector**
2) Sphere Decoder (5)

- Symbols-reordered K-Best
  - Schnorr-Euchner strategy [WMPF03]
  + Early termination of the tree search
  - Maximal complexity remains unchanged

- Layers-reordered K-Best [WTCM02]
  - ZF-ordering, re-order antennas by reducing SNR [WBKK03]
  - MMSE-ordering, re-order antennas by reducing SINR [WBKK03]
  + Combats errors propagation
  - Still not the ML diversity for high order constellations, $K$ must be chosen very large for low SNR symbols and would be chosen small for high SNR symbols
2) Sphere Decoder (6)

- MIMO-SM 4x4, Rayleigh channel, QPSK/16QAM
- QRD-based K-Best, SQRD-based K-Best
2) Sphere Decoder (6)

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2) Sphere Decoder (6)

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![Graphs showing BER vs SNR for complex Rayleigh 4x4 detectors performances, 10,000 runs.](image)
2) Sphere Decoder (7)

- Dynamic K-Best
  - Use larger $K$ in early stages and smaller $K$ in later stages [LW08], particularly efficient with SQRD
  - Performance: Avoid missing the ML solution in the first layers (most likely case of global error), Reduced complexity
  - How to set $K$, Still too complex for high order constellations

- Particular case: Fixed-Throughput Sphere-Decoder
  - Full-ML at $k$ top layers, Linear Equalizer (LE) at $n_T-k$ bottom layers [BT08]

3) Lattice-Reduction (1)

- General principle of Lattice-Reduction-Aided algorithms
  - Lattice definition: \( \mathcal{L} = HZ_C^{nT}, Z_C = Z + jZ \)
    - \( Z \) is the set of integers
    - \( H = [h_1, \ldots, h_n] \) is a generator basis
  - Interest: a basis is not unique
    - \( y = Hx + n \) rewrites \( y = HTT^{-1}x + n = H_{\text{red}}z + n \).
      Why not realizing equalization or detection through a better conditioned matrix \( H_{\text{red}} \)?

- What is a better conditioned matrix?
  - Shorter, more orthogonal
3) Lattice-Reduction (2)

- Lattice-Reduction algorithms
  - Korkine-Zolotareff
  - Lenstra-Lenstra-Lovasz (LLL) [LLL82]
  - Complex LLL (CLLL) imply complexity reduction [GLM06]
  - Seysen [Sey93]
  - SQRD-based LLL less complex, Seysen may be parallelized
3) Lattice-Reduction (3)

- **LLL [LLL82]**
  - **Orthogonality condition**
    \[ |\mu_{i,j}| < 1/2 \]  
    \[ \mu = \frac{\langle h_i, h_j \rangle}{\langle h_i, h_j \rangle} \]  
    Size reduction operation makes vectors shorter and more orthogonal
  - **Short norms condition**
    \[ ||h_i||^2 + \mu_{i,i-1} ||h_{i-1}||^2 > \delta ||h_{i-1}||^2 \]  
    Swapping operation if condition violated
  - **\( T \) unimodular** (contains Gaussian integers \( \mathbb{Z}_C \) and \( |\text{det}\{T\}|=1 \))
    The reduced constellation \( z \in \mathbb{Z}_C^{n_T} \)
    The \( n_T \)-parallelootope \( n_T \)-volume formed by the basis remains unchanged (same channel impact (SNR))
  - Worst case polynomial complexity, complexity reduction through the (necessary) SQRD starting point, no channel knowledge at transmitter
  - Random complexity, iterative algorithm

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3) Lattice-Reduction (4)

- General principle of Lattice-Reduction-Aided algorithms
  - \( \mathbf{v}_1 = [7, 6]^T \)
  - \( \mathbf{v}_2 = [10, 8]^T \)
3) Lattice-Reduction (4)

- **General principle of Lattice-Reduction-Aided algorithms**
  - \( \mathbf{v}_1 = [7, 6]^T \)
  - \( \mathbf{v}_2 = [10, 8]^T \)
- **Size reduction**
  - \( \mathbf{v}_1 = [7, 6]^T \)
  - \( \mathbf{v}_2 = [3, 2]^T \)
3) Lattice-Reduction (4)

- General principle of Lattice-Reduction-Aided algorithms
  - $\mathbf{v}_1 = [7, 6]^T$
  - $\mathbf{v}_2 = [10, 8]^T$
- Size reduction
  - $\mathbf{v}_1 = [7, 6]^T$
  - $\mathbf{v}_2 = [3, 2]^T$
- Swapping
  - $\mathbf{v}_1 = [3, 2]^T$
  - $\mathbf{v}_2 = [7, 6]^T$
3) Lattice-Reduction (4)

- General principle of Lattice-Reduction-Aided algorithms
  - $\mathbf{v}_1=\begin{bmatrix} 7, 6 \end{bmatrix}$
  - $\mathbf{v}_2=\begin{bmatrix} 10, 8 \end{bmatrix}$
- Swapping
  - $\mathbf{v}_1=\begin{bmatrix} 3, 2 \end{bmatrix}$
  - $\mathbf{v}_2=\begin{bmatrix} 7, 6 \end{bmatrix}$
- Size reduction
  - $\mathbf{v}_1=\begin{bmatrix} 3, 2 \end{bmatrix}$
  - $\mathbf{v}_2=\begin{bmatrix} 1, 2 \end{bmatrix}$
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- General principle of Lattice-Reduction-Aided algorithms
  - \( \mathbf{v}_1 = [7, 6]^T \)
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- Size reduction
  - \( \mathbf{v}_1 = [3, 2]^T \)
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- Size reduction
  - \( \mathbf{v}_1 = [1, 2]^T \)
  - \( \mathbf{v}_2 = [2, 0]^T \)

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- General principle of Lattice-Reduction-Aided algorithms
  - $\mathbf{v}_1 = [7, 6]^T$
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- Size reduction
  - $\mathbf{v}_1 = [1, 2]^T$
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- Swapping
  - $\mathbf{v}_1 = [2, 0]^T$
  - $\mathbf{v}_2 = [1, 2]^T$

- $(\mathbf{v}_1, \mathbf{v}_2)$ is LLL-reduced
3) Lattice-Reduction (5)

- Impact on detection step

\[ x \rightarrow H \rightarrow n \rightarrow y \rightarrow H^\dagger \rightarrow \mathcal{F} \rightarrow x_{est} \]

- Quantification [Bar08]
- Non-existing symbols vectors
3) Lattice-Reduction (6)

- MIMO-SM 4x4, Rayleigh channel, QPSK/16QAM
- LRA-ZF, LRA-MMSE, LRA-MMSE Extended [WBKK04]
- LRA-SIC, LRA-OSIC [WBKK04]

+ ML diversity, worst case polynomial complexity, independent of SNR
- Additional complexity, SNR offset

QPSK

16QAM
4) Lattice-Reduction-Aided Sphere Decoder (1)

- Principle of combination of both
  - Get the LRA technique diversity and reduce the SNR offset through a neighborhood study

\[ x_{LRA-SD} = \arg\min\ |y - HTT^{-1}x|^2 = \arg\min\ |y - H_{red}z|^2 \]

- Problem of neighborhood generation: \[ Z_{all} = T^{-1}X_{all}, \] ML complexity
4) Lattice-Reduction-Aided Sphere Decoder (2)

- Reduced constellation neighborhood study algorithm
  - Qi, Holt algorithm [QH07]
  - Shift-scale-normalization: \( y' = (y + Hd)/2, \ d = 1/2 \)
  - Shift-scale-normalization: \( x' = (x + d)/2, \ z = T^{-1}x' \)
  - \( x_{LRA-SD} = \arg\min | | Q_{red}^H y' - R_{red} z | |^2 \)

- Neighborhood exploration through a predetermined set of displacements around SIC solution: \( [\delta_1, ..., \delta_N] \), \( N > K \)

+ Improve performances (exploits reduced lattice advantages concerning channel conditions) with low number of candidates

- No limitation of number of explored symbols (infinite lattice), \( z_{est} \) could give non-existing \( x_{est} \) in the original constellation (increase complexity or decrease performance)
4) Lattice-Reduction-Aided Sphere Decoder (3)

- Reduced constellation neighborhood study algorithm
- Candidate generation limitation [RGAV09]

\[ x'_\text{min/\text{max}} \text{ and } T \text{ are known} \Rightarrow z_{\text{min/\text{max}}} \text{ are known} \]

\[
z_{\text{max}}(l) = x'_{\text{max}} \Sigma (T^{-1})(l,:) > 0 + x'_{\text{min}} \Sigma (T^{-1})(l,:) < 0,
\]

\[
z_{\text{min}}(l) = x'_{\text{min}} \Sigma (T^{-1})(l,:) > 0 + x'_{\text{max}} \Sigma (T^{-1})(l,:) < 0
\]

- Complexity reduction without performance loss
4) Lattice-Reduction-Aided Sphere Decoder (4)

- Original constellation neighborhood study algorithm
  - LRA-ZF centered SD [ZM07]

\[
x_{\text{est}} = \arg\min \left| \left| y_{\text{LRA-ZF}} - z \right| \right|^2 = \arg\min \left| \left| y_{\text{LRA-ZF}} - T^{-1}x \right| \right|^2 = \arg\min \left| \left| Q_{T^{-1}} y_{\text{LRA-ZF}} - R_{T^{-1}}x \right| \right|^2
\]

+ Reduced complexity (Although the needed QRD of \( T^{-1} \) needed), Avoid non-existing symbols vectors

- Performance (does not exploit reduced lattice advantages concerning channel conditions)
4) Lattice-Reduction-Aided Sphere Decoder (5)

- MIMO-SM 4x4, Rayleigh channel, QPSK/16QAM
- LRA-KBest in original/reduced constellation

+ Performances are independent of constellation order

- Benefits are limited in QPSK case, less sensitive to ill-conditioned channel and many inexistent symbols vectors [ZG06]
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Conclusion and further studies

- Hard-Decision performance is near-ML
- Full detector computational complexity study is necessary

- Soft-Decision extension
- Closed-loop and OFDM case calibration
- Throughput objectives of LTE-A norm must be shown to be reached
Q&A and discussion

➢ Thank you for your attention

➢ Questions?
References (1)


References (2)


[GLM06] Y.H. Gan, C. Ling, and W.H. Mow, " Complex Lattice Reduction Algorithm for Low-Complexity MIMO Detection," ...  


