

Peak-Factor reduction in O.F.D.M. by Reed-Müller channel coding : a new soft decision decoding algorithm

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Abstract-This paper presents a soft decision decoding based on Chase's algorithm applied to Reed-Müller codes for peak-to-Mean Power Ratio (PMEPR) reduction and error correction. This scheme is applied to the OFDM (Orthogonal Frequency Division Multiplexing) modulated signal to reduce its PMEPR and to have correction capacities. We first obtain a 1.2 dB coding gain at BER=10⁻⁴ between hard and soft decoding and about 4 dB between the soft decoding and the uncoded system. Next, by modeling nonlinear TWTA power amplifiers with AM/AM and AM/PM distortions, we obtain similar results and note improving BER performances compared to BCH codes with the same parameters.

Index Terms-OFDM, Reed-Müller coding, peak-factor, non-linear amplifier.

I. BACKGROUND

A. The OFDM peak-factor challenge

The advantages of the OFDM modulation are now well known. Based on the concept of transmitting the data in parallel QAM modulated sub-carriers using frequency division multiplexing, the OFDM modulation requires no adaptation to instantaneous channel responses and is easy to implement thanks to the progresses in the electronics field these recent years [1]-[3]. For instance, the DAB (Digital Audio Broadcasting) and the DVB-T (Digital Video Broadcasting by terrestrial networks) technologies are based on OFDM. But the major drawback of OFDM is the high peak factor of the modulated signal (defined as the instantaneous to the mean power ratio, designed as the PMEPR). This could exhibit distortions and intermodulation noise when the signal is passing through nonlinear circuits like power amplifiers at the emission side.

Let N be the number of carriers, C_k , $k=\{0,\dots,N-1\}$, the complex information symbols vector and T_s the OFDM symbol length. The complex envelope of the modulated signal is :

$$S(t) = \sum_{k=0}^{N-1} C_k e^{2i\pi k \frac{t}{T_s}} \quad (1)$$

and the PMEPR of $S(t)$ is defined as:

$$PMEPR[S(t)] = \frac{\max_{0 \leq t \leq T_s} |S(t)|^2}{\frac{1}{T_s} \int_0^{T_s} |S(t)|^2 dt} \quad (2)$$

We can show, that without processing, $PMEPR_{S(t)} = 10 \log(N)$ (dB), what is high if N is large and makes the signal $S(t)$ susceptible to nonlinear or clipping distortions, as the signal peaks may occasionally thrust into the saturation region of the power amplifier. The PMEPR characterises the signal's fluctuations and the idea is to reduce the dynamic of $S(t)$, i.e. its PMEPR.

B. The Reed-Müller coding scheme

A novel idea based on Reed-Müller codes [4] reduces the PMEPR of a signal on the one hand and exploits on the other hand the correction capacity of the code to correct errors : the PMEPR is reduced to a level of 3 dB, theoretically apart from N . This coding is based on the following theorem which is the link between the OFDM modulation and the Reed-Müller codes : let $RM(1,m)$ be a first order Reed-Müller code of parameter m and G its generator matrix of rows x_i . For a 2^h -PSK modulation, the key theorem is as follow [2] :

$$D = 2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + G^T u \quad (3)$$

is a 3 dB PMEPR vector for any permutation π of the symbols and for any information vector $u \in \{0,1,\dots,2^h\}^m$. $D = (d_0, \dots, d_k, \dots, d_{2^h-1})$ is called a Golay sequence. Then, using the vector

$$C_k = e^{2i\pi \frac{d_k}{2^h}}$$

and the relation (1), $S(t)$ is a 3 dB PMEPR signal.

The code rate τ depends on m and then on $N=2^m$ (the number of carriers) according to :

$$\tau = \frac{\left\lfloor \log_2 \frac{m!}{2} \right\rfloor + h(m+1)}{h2^m} \quad (4)$$

and the Hamming distance d_H of the code is :

$$d_H = 2^{m-2} = \frac{2^m}{4} = \frac{N}{4} \quad (5)$$

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Nevertheless, from (4), the code rate decreases highly with N (or with m) and Fig. 1 shows its evolution for a BPSK modulation (for higher constellation sizes, the code rate is still more weak).

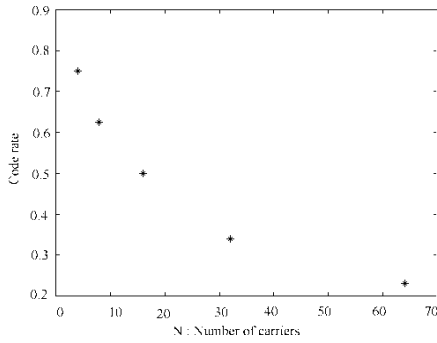


Fig.1 : evolution of the code rate for a BPSK modulation

According to (5), the code has correction capacity if $N \geq 16$. This implies a code rate $\tau \leq 0.5$ in BPSK and $\tau \leq 0.4$ in QPSK. These parameters (N=16, QPSK) have been chosen for the computer simulations.

Fig. 2 presents the OFDM block diagram used. Nonlinear power amplifiers are used in the last part of the paper.

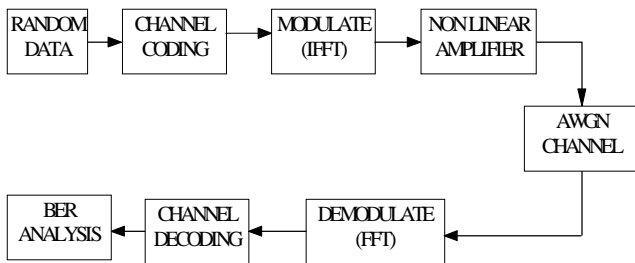


Fig.2 : OFDM block diagram

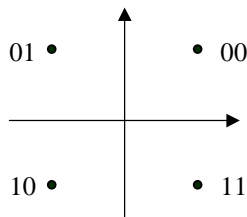
II. SOFT DECISION DECODING

A. The reliability function

According to the relation (1), the modulated symbol C_k is :

$$C_k = e^{\frac{2i\pi d_k}{2^h}}$$

for a Golay sequence $D=(d_0, \dots, d_2^m \cdot 1)$. Then, the associated constellation for h=2 is (with a $\pi/4$ rotation) :



Then, the coding theory of combined OFDM-Reed-Müller imposes a symbol mapping which does not respect a Gray coding. Moreover, the codewords are too much important to use a soft MVP decoding. Then, we have set Chase's algorithm : according

to a reliability function, we select the least reliable symbols and replace them by their nearest neighbours.

Let S be the received symbol, we first make a hard decision to S and decide C_k as the emitted symbol. For each symbol $C_l \neq C_k$ we calculate a reliability function :

$$\Lambda_{lk, l \neq k} = \text{Log} \left[\frac{\text{Pr}(C_k / S)}{\text{Pr}(C_l / S)} \right]$$

In the constellation, we define $S(x,y)$ and $C_l(x_l, y_l)$. With independent AWGN(0, σ^2) in X and Y and according to Bayes' theorem,

$$\Lambda_{lk, l \neq k} = \text{Log} \left[\frac{\text{Pr}(x, y / x_k, y_k)}{\text{Pr}(x, y / x_l, y_l)} \right]$$

where

$$\text{Pr}(x / x_k) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_k)^2}{2\sigma^2}}$$

Then, we can show that :

$$\Lambda_{lk, l \neq k} = \frac{d_l^2 - d_k^2}{2\sigma^2} \quad (6)$$

where d_l and d_k are respectively the euclidean distance between S and C_l and between S and C_k . For each symbol, we calculate 3 associated reliable functions and affect it the minimum of the three values. Then, in the all sequences, we select the symbols which have the minimum affected values (which correspond to the least reliable symbols) and change them with their nearest neighbours in the constellation.

B. Simulation results

So as to perform the soft decoding, we have to decide how many L least reliable symbols we choose to apply Chase's theorem. According to the method 2 [6], we take L=2 This implies a increasing complexity of 4^2+1 decoding compared to hard decoding.

We have computed an OFDM communication system for an AWGN channel (in this case with no nonlinear power amplifier). The simulation results are shown in Fig. 3.

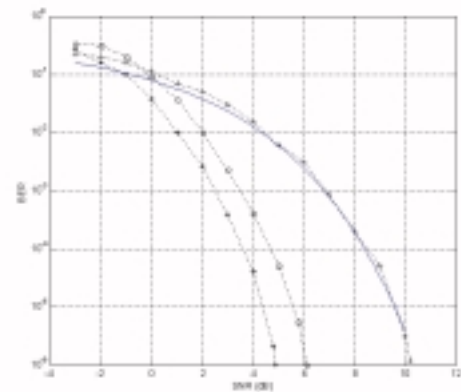


Fig. 3 : hard and soft decoding in AWGN channel

- * -- : soft decoding
- o -- : hard decoding
- : uncoded QPSK

We observe a 1.2 dB coding gain at BER=10⁻⁴ between hard and soft decoding and about 4 dB between soft decoding and the uncoded system. The parameters used for the simulations are :

- number of carriers : N=16 (m=4)
- constellation size : h=2 (QPSK)
- code rate : $\tau = 0.40$

III. NONLINEAR EFFECTS

A. Introduction

Power amplifiers have nonlinear characteristics in amplitude and phase (known as AM/AM and AM/PM transfer characteristics respectively). Assuming a complex baseband representation of the modulated signals and an input signal $S_{in}(t)$, the input/output relationship of the amplifier can be expressed as :

$$S_{out}(t) = f(|S_{in}(t)|)e^{i[\varphi_{in}(t)+g(|S_{in}(t)|)]} \quad (6)$$

where $S_{out}(t)$ is the output of the complex signal. $f(\cdot)$ and $g(\cdot)$ describe AM/AM and AM/PM distortions. In this paper, we have focused on the TWTA (Traveling-Wave Tube Amplifier) model for which :

$$f(A) = \frac{A}{1 + \alpha A^2} \quad g(A) = \frac{\beta_0 A^2}{1 + \beta_1 A^2} \quad (7)$$

A is the instantaneous envelope amplitude of the input signal. f has maximum value for $A_{max}^2 = 1/\alpha$ [5]. For a N length uncoded sequence (with no particular propriety) whose components are c_i , $i \in \{0, \dots, N-1\}$, the peak amplitude of the OFDM associated envelope signal is equal to $Nmax(|c_i|)$. In this paper, we have set the condition $A_{max} = Nmax(|c_i|)$. The advantage for using a Reed-Müller code to generate Golay sequences is to have a peak amplitude value after the IFFT of only $\sqrt{2N} max(|C_i|)$ instead of $Nmax(|C_i|)$. Fig.4 and 5 are respectively the AM/AM and AM/PM TWTA nonlinear distortions used in this paper.

We have chosen f and g parameters as follow : $\alpha = 0.00195$, $\beta_0 = \pi/12$ and $\beta_1 = 0.5$ [5]. The maximum constellation deviation caused by AM/PM distortions is then β_0/β_1 , i.e. $\pi/6$, according to (7).

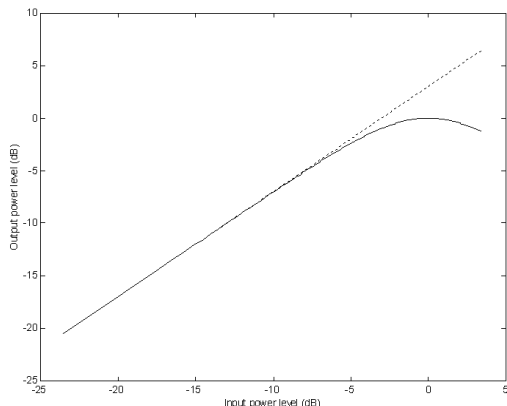


Fig. 4 : AM/AM TWTA transfer curve

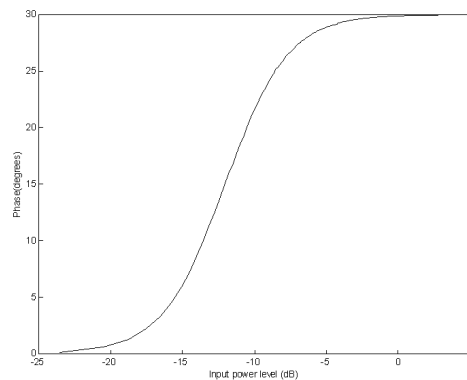


Fig. 5 : AM/PM TWTA transfer curve

B. Simulation results with nonlinear power amplifiers

For the simulations, we have chosen N=16 (m=4) and a QPSK modulation on each carrier (h=2). Then the code rate is 0.41. The soft decoding parameters used are the same as in Part II.

Fig. 6 and 7 show the BER performance results respectively for AM/AM and AM/PM distortions, both in presence of AWGN. We can observe about 2 dB improvement between hard and soft decoding even with a constellation rotation caused by AM/PM distortions.

Fig. 8 shows a comparison between BCH and Reed-Müller codes in case of nonlinear conditions. The BCH code used has similar parameters so as to be compared to Reed-Müller's performances. We have taken BCH(15,5,7). Once again, thanks to a 3 dB PMEPR, the Reed-Müller codes have a better BER than BCH codes, even if the correction capacities are similar.

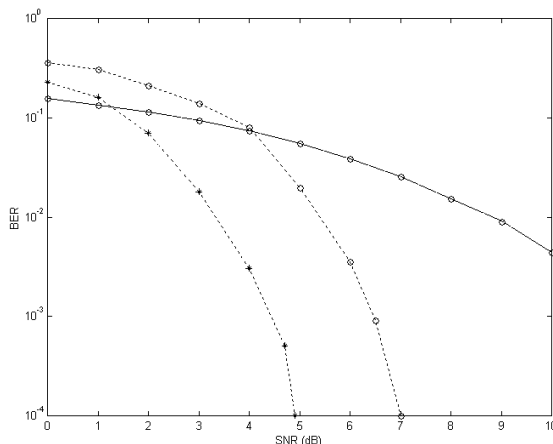


Fig. 6 : hard, soft decoding and uncoded system for AM/AM TWTA distortions

- * --- : soft decoding
- o --- : hard decoding
- o — : uncoded system

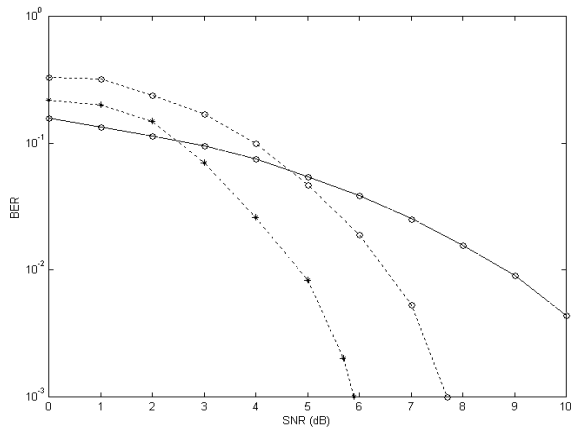


Fig. 7 : hard, soft decoding and uncoded system for AM/PM TWTA distortions

--- * --- : soft decoding
 --- o --- : hard decoding
 — o — : uncoded system

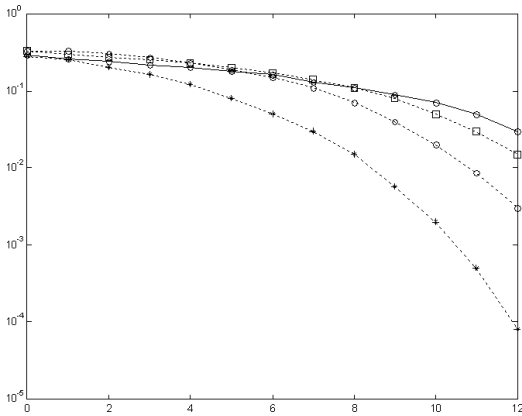


Fig. 8 : hard, soft, uncoded and BCH(15,5,7) for AM/PM distortions :

--- * --- : soft decoding
 --- o --- : hard decoding
 --- □ --- : BCH code (hard decoding)
 — o — : uncoded system

IV. CONCLUSION

This paper aims to present the performances of the Reed-Müller codes applied to OFDM for peak-factor reduction. We have developed a soft decision decoding based on Chase's algorithm. A 1.2 dB coding gain is obtained compared to the hard decoding and 4 dB compared to an uncoded system at $BER=10^{-4}$. Similar results are obtained by modelling AM/AM and AM/PM nonlinear TWTA distortions. Next, we observe better results compared to BCH codes with the same parameters, caused by the fact that the codewords haven't a 3 dB PMEPR.

V. REFERENCES

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