A soft decision decoding of product BCH and Reed-Müller codes for error control and peak-factor reduction in OFDM

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Abstract - Orthogonal Frequency Division Multiplexing (OFDM) is a very attractive modulation scheme for data transmission in multipath channels. But the high Peak-to-MEan Power Ratio (PMEPR) of the modulated signal is a major drawback and makes the use of some circuits very tricky : for instance, power amplifiers at the emission side could have nonlinear characteristics and a high PMEPR could exhibits intermodulation noise and distortions.

A very recent theory based on Reed-Müller codes controls the PMEPR and exploits the correction capacity of the code to correct error propagation : the PMEPR is reduced to a level of 3 dB and the distance of the code is the quarter of the codeword (in this specific application of Reed-Müller and OFDM). But the correction capacity is limited and have to be balanced with the code rate.

This paper first presents a product coding scheme based on BCH and Reed-Müller codes to improve error correction and to keep the PMEPR propriety of the modulated signal. Next, we present a soft decision decoding based on Chase’s algorithm associated to Reed-Müller codes. Then, we analyse the BER performances of both hard and soft decoding associated to the product coding scheme modelling AWGN and nonlinear TWTA power amplifiers. With this soft decision decoding, we obtain a 1.5 dB coding gain compared to hard decoding when using product codes and about 5 dB coding gain compared to an uncoded system, at BER=10^{-4}. Balanced with the global code rate, BCH(255,231,7) with Reed-Müller codes have higher coding gain compared to other product codes. Nonlinear distortions degrade BER performances and the coding gain is still about 2 dB at BER=10^{-4}.

The applications concerns high data rate and broadband transmissions using OFDM where power efficiency is a major challenge.

Key-Words - OFDM, peak-factor, coding, Reed-Müller codes, soft decoding, nonlinear amplifiers.

1 Background

1.1 The OFDM modulation

The advantages of the OFDM modulation are now well known. Based on the concept of transmitting the data in parallel QAM modulated sub-carriers using frequency division multiplexing, the OFDM modulation requires no adaptation to instantaneous channel responses and is easy to implement thanks to the progresses in the electronics field these recent years [2]-[4]. The DAB (Digital Audio Broadcasting) and the DVB-T (Digital Video Broadcasting by terrestrial networks) technologies are based on OFDM. But the major drawback of OFDM is the high peak-factor of the modulated signal (defined as the instantaneous power to the mean power ratio, designed as the PMEPR). This could exhibits distortions and intermodulation noise when the signal is passing through nonlinear circuits like power amplifiers. Let N be the number of carriers, C_k, $k=\{0,...,N-1\}$, the complex information symbols vector and T_S the OFDM symbol length. The complex envelope of the modulated signal is :

$$S(t) = \sum_{k=0}^{N-1} C_k e^{2i\pi k \frac{t}{T_S}} \qquad (1)$$

and the PMEPR of $S(t)$ is :

$$PMEPR\ [S(t)] = \frac{\max\limits_{0 \le t \le T_S} |S(t)|^2}{\frac{1}{T_S}\int_0^{T_S} |S(t)|^2\, dt} \qquad (2)$$

We can show, that without processing, $PMEPR_{S(t)}=10Log(N)$ (dB). The PMEPR characterises the signal's fluctuations and the idea is to reduce the dynamic of $S(t)$, i.e. its PMEPR.

1.2 The Reed-Müller coding scheme

A novel idea based on Reed-Müller codes [4] reduces the PMEPR of a signal on the one hand and exploits on the other hand the correction capacity of the code to correct errors : the PMEPR is reduced to a level of 3 dB, theoretically apart from N. This coding is based on the following theorem which is the link between the OFDM modulation and the Reed-Müller codes : let RM(1,m) be a first order Reed-Müller code of parameter m and G its generator matrix of rows x_i .

For a 2^h-PSK modulation, the key theorem is as follow [5] :

$$D=2^{h-1}\sum_{k=1}^{m-1} x_{\pi(k)}x_{\pi(k+1)} + G^T u \quad (3)$$

is a 3 dB PMEPR vector for any permutation π of the symbols and for any information vector $u \in \{0,1,...,2^h\}^m$. $D=(d_0,\ldots,d_k,\ldots,d_{2^m-1})$ is called a Golay sequence. Then, using the vector $C_k = e^{(2i\pi d_k/2^h)}$ and the relation (1), $S(t)$ is a 3 dB PMEPR signal. The code rate τ depends on m and then on $N=2^m$ (the number of carriers) according to :

$$\tau = \frac{\left|Log_2 \frac{m!}{2}\right| + h(m+1)}{h2^m} \quad (4)$$

and the Hamming distance d_H of the code is :

$$d_H = 2^{m-2} = \frac{2^m}{4} = \frac{N}{4} \quad (5)$$

Nevertheless, from (4), the code rate decreases highly with N (or with m) and Fig.1 shows its evolution for a BPSK modulation (for higher constellation sizes, the code rate is still more weak) :

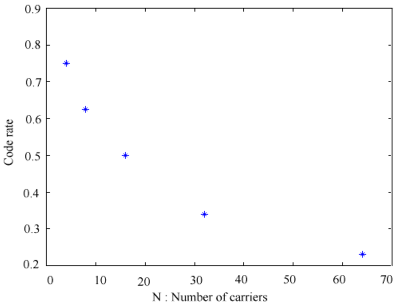


Fig.1 : code rate for a BPSK modulation

According to (5), the code has correction capacity if N ≥16. This implies a code rate $\tau \leq 0.5$ in BPSK and $\tau \leq 0.4$ in QPSK. These parameters (N=16, QPSK) have been chosen for the computer simulations. Fig.2 presents the OFDM block diagram used.

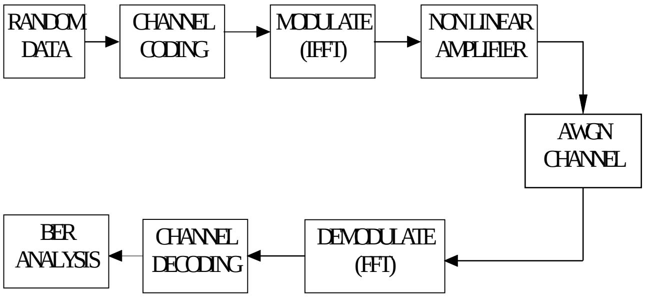


Fig. 2 : the OFDM system diagram

2 Properties of product codes

2.1 Introduction

The principle of product codes is to combine two codes $C_1(n_1,k_1,d_1)$ and $C_2(n_2,k_2,d_2)$. Placing the $k_1 \times k_2$ information bits in an array, the coding is as follow : the k_1 rows are coded with C_2 and the n_2 columns are coded with C_1. The result code $C = C_1 \otimes C_2$ has parameters :

- $n = n_1 n_2$
- $k = k_1 k_2$
- $d = d_1 d_2$
- $R = R_1 R_2$ where R is the code rate of C and R_i the code rate of C_i. The idea is to build more powerful codes with large distances.

2.2 Application to OFDM

The idea is to associate BCH and Reed-Müller codes for both error correction and peak-factor reduction. In our application, Reed-Müller code (RM) parameters are n_1= 13, k_1= 32 and d_1= 4. For equal BCH error capacities, we have computed product codes performances for :

P_1 = BCH(31,16,7) ⊗ RM,
P_2= BCH(63,45,7) ⊗ RM,
P_3 = BCH(127,106,7) ⊗ RM,
P_4 = BCH(255,231,7) ⊗ RM,

Results are shown in Fig. 3. The BER is first estimated for hard Reed-Müller decoding in an AWGN channel. The number of carriers of the OFDM process is equal to 16, for a QPSK modulation on each carrier.

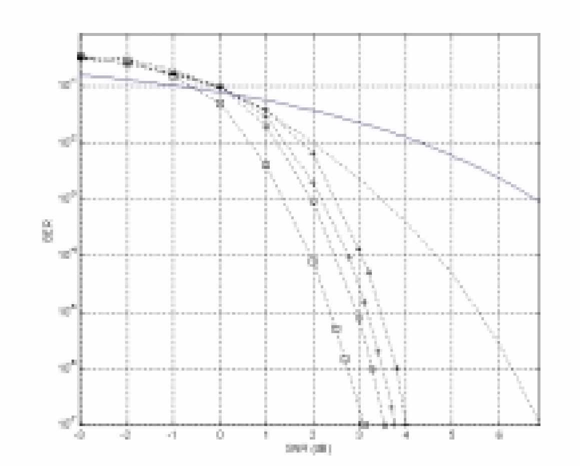


Fig. 3 : BER for Reed-Müller hard decoding (with AWGN)

... □ ... : P_1 = BCH(31,16,7) ⊗ RM,
... o ... : P_2 = BCH(63,45,7) ⊗ RM,
…+… : P_3 = BCH(127,106,7) ⊗ RM,
…✻… : P_4 = BCH(255,231,7) ⊗ RM,
------ : RMGS (hard decisions)

2.3 Comments

From Fig.3, for the same error correction capacity (here d=7), we can first see that the more the BCH code rate is close to 1, the more its associated product code with Reed-Müller has a bad BER. If we have a look to P_1, P_2, P_3 and P_4 code rates, they are respectively equal to 0.2, 0.29, 0.34 and 0.37. So, the code rate has to be balanced with the BER of the product code.

To improve the BER of these product codes, we have set a soft decision decoding for the Reed-Müller codes.

3 A soft decision decoding for Reed-Müller codes

3.1 Introduction

According to the relation (1), the modulated symbol C_k is :

$$C_k = e^{\frac{2i\pi d_k}{2^h}}$$

for a Golay sequence $D=(d_0,\ldots,d_{2^m-1})$. Then, the mapping of the OFDM-Reed-Müller coding scheme does not respect a Gray coding. Moreover, the all codewords makes the use of the MAP (Maximum A-posteriori Probability) soft decision too complex. That’s why a soft decision decoding using Chase's algorithm has been developed. This decoding selects the least reliable symbols according to a reliability function and replaces them by their nearest neighbours in the constellation. Then, it decodes the new sequences.

3.2 The reliability function

Let S be the received symbol, we first make a hard decision to S and decide C_k as the emitted symbol [1]. For each symbol $C_l \neq C_k$ we calculate a reliability function :

$$\Lambda_{lk,l\neq k} = Log\left[\frac{\Pr(C_k/S)}{\Pr(C_l/S)}\right] \quad (6)$$

where

$$\Pr(x/x_k) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_k)^2}{2\sigma^2}} \quad (7)$$

In the constellation, we define $S(x,y)$ and $C_l(x_l,y_l)$. With independent AWGN$(0,\sigma^2)$ in X and Y and according to Bayes 's theorem, we can show that :

$$\Lambda_{lk,l\neq k} = Log\left[\frac{\Pr(x,y/x_k,y_k)}{\Pr(x,y/x_l,y_l)}\right] = \frac{d_l^{\,2} - d_k^{\,2}}{2\sigma^2} \quad (8)$$

where d_l and d_k are respectively the euclidean distance between S and C_l and between S and C_k. For each symbol, we calculate 3 associated reliable functions and affect it the minimum of the three values. Then, in the all sequences, we select the symbols which have the minimum affected values (which correspond to the least reliable symbols) and change them with their nearest neighbours in the constellation.

For the computer simulations, we have chosen N=16 (m=4) and a QPSK modulation on each carrier (h=2). So as to perform the decoding, we have to decide how many L least reliable symbols we choose to apply Chase's theorem. According to the method 2 [7], we have taken L=4. This implies a increasing complexity of 4^2+1 decoding compared to hard decoding.

3.3 Simulation results of product codes with a soft decision decoding associated to Reed-Müler codes

Results are shown in Fig.4. The soft decoding is applied to the Reed-Müller codes. The channel is AWGN and the BER is estimated after the BCH decoding.

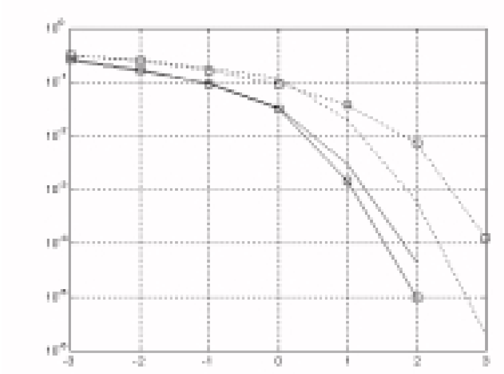


Fig. 4 : BER for BCH and Reed-Müller product codes with hard (P_1 and P_2) and soft (P_3 and P_4) decoding

... … : P_1 = BCH(255,231,7) ⊗ RM,
……. : P_2 = BCH(31,16,7) ⊗ RM,
— — : P_3 =BCH(255,231,7) ⊗ RM,
—— : P_4 = BCH(31,16,7) ⊗ RM,

We can see that the coding gains between hard and soft decoding is more important for BCH(255,231,7) ⊗ RM than for BCH(31,16,7) ⊗ RM. They are respectively equal to 1.5 dB and 0.5 dB at BER=10^{-4}. But even if P_4 BER performances are less significant compared to P_3 , these results have to be balanced with the global code rate of the product code : for P_4, the code rate is equal to 0.209 and equal to 0.368 for P_3. So the use of a soft decoding associated to BCH(255,231,7) ⊗ RM product code seems to be a good compromise.

4 Nonlinear amplifier models

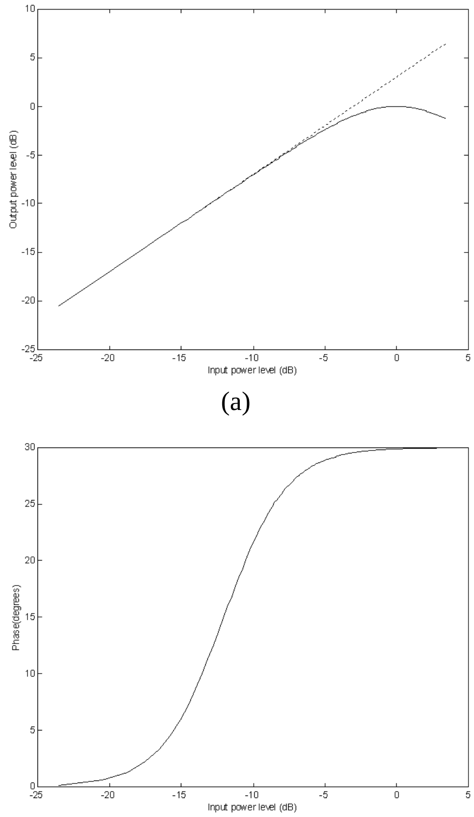
We have used nonlinear characteristics to focus on the PMEPR benefits of the Reed-Müller codes towards power efficiency. Power amplifiers have nonlinear characteristics in amplitude and phase (known respectively as AM/AM and AM/PM transfer distortions). Assuming a complex baseband representation of the modulated signals and an input signal $S_{in}(t)$, the input/output relationship of the amplifier can be expressed as :

$$S_{out}(t) = f(|S_{in}(t)|)e^{i[\varphi_{in}(t)+g(|S_{in}(t)|)]} \qquad (9)$$

where $S_{out}(t)$ is the output of the complex signal. $f(.)$ and $g(.)$ describe AM/AM and AM/PM distortions. In this paper, we have focused on the TWTA (Traveling-Wave Tube Amplifier) model where :

$$f(A) = \frac{A}{1+\alpha A^2} \quad ; \quad g(A) = \frac{\beta_0 A^2}{1+\beta_1 A^2} \qquad (10)$$

A is the instantaneous envelope amplitude of the input signal. f has maximum value for $A_{max}^2 = 1/\alpha$ [6].



(a)

(b)

Fig. 5 : AM/AM (a) and AM/PM (b) TWTA transfer curves

For a N length uncoded sequence (with no particular propriety) whose components are c_i, $i \in \{0,\dots,N-1\}$, the peak amplitude of the OFDM associated envelope signal is equal to $N max(|c_i|)$. In this paper, we have set the condition $A_{max}=N max(|c_i|)$. The advantage of using a Reed-Müller code to generate Golay sequences is to have a peak amplitude value after the IFFT of only $\sqrt{2N}\, max(|c_i|)$ instead of $N max(|c_i|)$. Fig.5 shows the AM/AM and AM/PM distortions used in this paper. We have taken **[**6**]**:

$$\alpha = 0.00195 \ ; \ \beta_0 = \pi/12 \ ; \ \beta_1 = 0.5$$

5 Simulation results for AM/AM and AM/PM distortions in an AWGN channel

Numerical values for β_0 and β_1 imposes a maximum angle constellation rotation of $\pi/6$ what explains the BER degradations for AM/PM distortions. The same comments are valuable for AM/AM distortions. The results are on Fig.6 (a) and (b) where we have used a BCH(31,16,7) for all the product codes simulations.

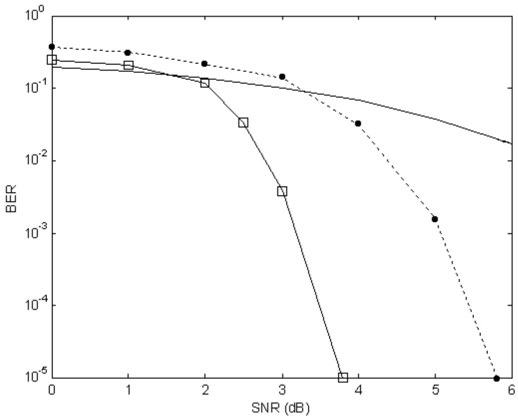


Fig 6 (a) : AM/AM + AWGN distortions

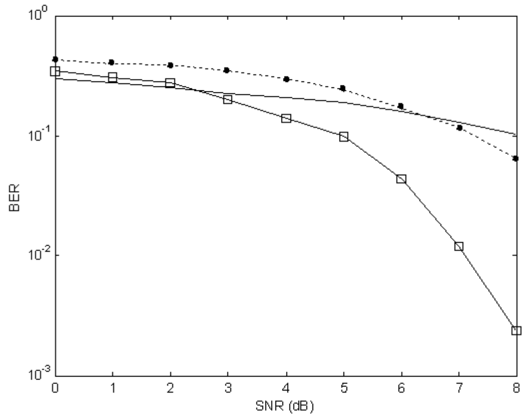


Fig. 6 (b) : AM/PM + AWGN distortions

…•… : BCH(31,16,7) ⊗ RM, hard decoding
—□— : BCH(31,16,7) ⊗ RM, soft decoding
——— : uncoded system

6 Conclusion

This paper aims to present a channel coding scheme to correct errors and to reduce the peak-factor of the OFDM modulated signal. We have developed a soft decision decoding based on Chase's algorithm and obtain a 1.5 dB coding gain at BER=10^{-4} between hard and soft decoding. By using BCH(255,231,7) and Reed-Müller codes as a product scheme, we have improved the BER performances and obtain a good compromise : a global code rate of 0.368 for a coding gain of about 1.5 dB between hard and soft decoding at BER=10^{-4}. But the number of carriers is only 16 what is quite weak for an OFDM application in severe channels with multipath or fading . We are carrying out our researches on this point.

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