Prioritized Throughput Maximization via Rate and Power Control for 3G CDMA: The Two Terminal Scenario

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Outline

- Motivation : Variable Spread Gain CDMA
- Simple Model for Single-Cell CDMA Data
- A Generalized Frame-Success Function
- Throughput equation is defined and optimization is performed
  - Interior “stationary” point (all partial derivatives set to zero) is sought. Second order conditions (SOC) are checked.
  - Boundary stationary point is sought in which bit rate of “important” user is pre-set as high as feasible. SOC are checked.
  - Boundary stationary point is sought in which bit rate of both users are pre-set at highest feasible level. SOC are checked.
- Related/future work

Spread Gain : \( G_i = \frac{R_C}{R_i} \) (Chip_rate / bit_rate) ; \( G_0 = \frac{R_C}{R_{\text{MAX}}} \)
\( \gamma_0 \) solves \( xf''(x)=f(x) \) ; \( \gamma_{00} \) solves \( x^2f'(x)=\frac{K(G_0)^2}{\beta} \) ; \( \beta \) : priority
Motivation: VSG-CDMA

- Modern (3G) wireless nets are expected to accommodate terminals operating at very different data transmission rates.
- Variable Spreading Gain CDMA can accommodate terminals operating at dissimilar bit rates.
- In a VSG CDMA system, chip rate is common, but each terminal's spreading (processing) gain is the ratio of the common chip rate to the terminal's bit rate.

Spread Gain: $G_i = \frac{R_C}{R_i}$ (Chip_rate / bit_rate); $G_0 = \frac{R_C}{R_{MAX}}$

$\gamma_0$ solves $xf'(x) = f(x)$; $\gamma_{00}$ solves $x^2f'(x) = \frac{K(G_0)^2}{\beta}$; $\beta$: priority
CDMA Single Cell Data Comm.

- \( N \) transceivers send data to a base station
- \( R_c : \) chip rate; \( R_i : \) data rate; \( G_i = R_c/R_i : \) Proc. Gain
- \( f_s(\gamma_i) : \) probability of correct reception of a packet
- \( \gamma_i = G_i \alpha_i \) is the SIR with \( \alpha_i \) the CIR given by

\[
\alpha_i = \frac{\sum_{j \neq i} h_j P_j + \sigma^2}{\sum_{i \neq j} Q_j + \sigma^2}
\]

\( h_i \) : “gain” factor

\( h_i P_i : = Q_i \) : received power

Spread Gain: \( G_i = R_c/R_i \) (Chip_rate / bit_rate); \( G_0 = R_C/R_{\text{MAX}} \)

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Spread Gain: \( G_i = \frac{R_C}{R_i} \) (Chip_rate / bit_rate); \( G_0 = \frac{R_C}{R_{\text{MAX}}} \)

\( \gamma_0 \) solves \( xf'(x) = f(x) \);

\( \gamma_{00} \) solves \( x^2f'(x) = K(G_0)^2/\beta \);

\( \beta \) : priority
Objective Function

- Want to maximize network weighted throughput: $\sum \beta_i R_i f_s(G_i \alpha_i)$
- $\beta_i$ is a priority weight
- Find for each active user, an optimal power level AND an optimal bit rate
- Power levels determined through optimal power ratios, $\alpha_i$ (CIR); and bit rates determined through optimal processing gains ($G_i$)
- CIR need to be constrained so that they lead to feasible power levels. For 2-user interference-limited system, $\alpha_1 = Q_1/Q_2 = 1/\alpha_2$ thus $\alpha_1 \alpha_2 = 1$
- Each $G_i$ must exceed certain $G_0 \geq 1$ ($R_i \leq R_M \leq R_c$)

Spread Gain: $G_i = R_c/R_i$ (Chip_rate / bit_rate) ; $G_0 = R_c/R_{MAX}$
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Optimization Model

Maximize \( \frac{f(G_1 \alpha_1)}{G_1} + \beta \frac{f(G_2 \alpha_2)}{G_2} \)

subject to \( \alpha_1 \alpha_2 = 1 \)
\( G_1 \geq G_0 \)
\( G_2 \geq G_0 \)

Spread Gain: \( G_i = \frac{RC}{R_i} \) (Chip_rate / bit_rate); \( G_0 = \frac{RC}{R_{MAX}} \)
\( \gamma_0 \) solves \( xf''(x)=f(x) \); \( \gamma_{00} \) solves \( x^2f'(x)=K(G_0)^2/\beta \); \( \beta \) : priority
First-Order Necessary Optimizing Cond.

Augmented Objective Function:

\[ \phi(G_1, G_2, \alpha_1, \alpha_2) = \frac{f(G_1 \alpha_1)}{G_1} + \beta \frac{f(G_2 \alpha_2)}{G_2} + \lambda(1 - \alpha_1 \alpha_2) + \mu_1(G_0 - G_1) + \mu_2(G_0 - G_2) \]

First-Order Necessary Optimizing Conditions (FONOC):

\[
\begin{bmatrix}
\frac{\gamma_1 f'(\gamma_1) - f(\gamma_1)}{G_1^2} - \mu_1 \\
\frac{\beta(\gamma_2 f'(\gamma_2) - f(\gamma_2))}{G_2^2} - \mu_2 \\
f'(\gamma_1) + \lambda \alpha_2 \\
\beta f'(\gamma_2) + \lambda \alpha_1
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

where \( \gamma_i = G_i \alpha_i \)

Spread Gain: \( G_i = \frac{R_C}{R_i} \) (Chip_rate / bit_rate) ; \( G_0 = \frac{R_C}{R_{\text{MAX}}} \)

\( \gamma_0 \) solves \( x f'(x) = f(x) \); \( \gamma_{00} \) solves \( x^2 f'(x) = K(G_0)^2/\beta \); \( \beta \) : priority
Interiors stationary point

- Seek a solution to FONOC in the interior of the feasible region; i.e., suppose $G_1 > G_0$, $G_2 > G_0$ (Lagrangian coefficients $\mu_1 = \mu_2 = 0$)
- This yields closed-form solution:
  \[ \alpha_1 = 1/\alpha_2 = \sqrt{\beta} \quad ; \quad G_1 \alpha_1 = G_2 \alpha_2 = \gamma_0 \]
- $\gamma_0$ solves $xf'(x) = f(x)$. It’s unique. (see fig.)
- Consistency requires that $G_1 = \gamma_0/\sqrt{\beta} > G_0$
- Second order conditions indicate this solution is always a “saddle point”
- This allocation is ‘fair’: both users enjoy same weighted throughput

Spread Gain: $G_i = R_c / R_i$ (Chip_rate / bit_rate); $G_0 = R_c / R_{\text{MAX}}$

$\gamma_0$ solves $xf'(x) = f(x)$; $\gamma_{00}$ solves $x^2f'(x) = K(G_0)^2/\beta$; $\beta$: priority
Asymmetric-rate boundary allocation

- Spread Gain: $G_i = \frac{R_C}{R_i}$ (Chip_rate/bit_rate) ; $G_0 = \frac{R_C}{R_{\text{MAX}}}$
- $\gamma_0$ solves $xf'(x)=f(x)$
- $\gamma_0$ solves $x^2f'(x) = K(G_0)^2/\beta$ ; $\beta$: priority

Seek a solution to FONOC on the boundary of the feasible region by supposing that $G_2 = G_0$ (“favorite” terminal operates at highest feasible bit rate) but that $G_1 > G_0$ (Lagrangian coeff. $\mu_1 = 0$).

- Solution exists whenever there is an $x$ satisfying $x^2f'(x)/f'(\gamma_0) = (G_0)^2/\beta$.

- The left-hand side of this equation is a “bell-shaped” function. Thus, if $(G_0)^2/\beta$ is “too large” no such $x$ exists. Otherwise, this equation has two solutions.

- Let $\gamma_{00}$ be the largest of the two values satisfying $x^2f'(x) = f'(\gamma_0)(G_0)^2/\beta$. All the optimizing values can be determined in terms of $\gamma_{00}$ and $\gamma_0$.

- $\gamma_{00}$ gives the optimal SIR of “favorite” user; i.e., $G_2 \alpha_2 = G_0 \alpha_2 = \gamma_{00}$. From this, $\alpha_2 = \gamma_{00}/G_0 = 1/\alpha_1$.

- The optimal SIR of less important user is $\gamma_0$ (preceding slide). This leads to $G_1 = \gamma_0 \gamma_{00} / G_0$. If this value does not exceed $G_0$ as was presumed, this allocation must be discarded. Thus $G_0 < \sqrt{(\gamma_0 \gamma_{00})}$.

- Second order conditions confirm that whenever this solution exists, it is a maximizer.
Form of $x^2 f'(x)$ and related functions

Scaled plots of a particular $f(x)$ [solid], $f'(x)$ [dotted], $xf'(x)$ [dashdot], and $x^2 f'(x)$ [dashed]

Spread Gain: $G_i = \frac{R_C}{R_i} \text{ (Chip_rate / bit_rate)}$; $G_0 = \frac{R_C}{R_{\text{MAX}}}$

$\gamma_0$ solves $xf'(x)=f(x)$; $\gamma_{00}$ solves $x^2 f'(x)=K(G_0)^2/\beta$; $\beta$ : priority
Spread Gain: $G_i = \frac{R_C}{R_i}$ (Chip_rate / bit_rate); $G_0 = \frac{R_C}{R_{\text{MAX}}}$

$\gamma_0$ solves $x f'(x) = f(x)$; $\gamma_{00}$ solves $x^2 f'(x) = K(G_0)^2 / \beta$; $\beta$: priority

“Greedy” Allocation

- Seek a solution to FONOC on the boundary of the feasible region by supposing that $G_2 = G_1 = G_0$ (both terminals operate at highest feasible bit rate)
- Solution always exists
- Second order conditions indicate that this solution may be a maximizer or a minimizer depending upon system parameters.
- When both terminals are equally important, equal-received power allocation ($\alpha_1 = \alpha_2 = 1$) satisfies FONOC. But this is a maximizer only when $G_0$ is “large enough”; i.e., it exceeds a threshold determined by frame-success function (the value at which $x f''(x)$ reaches maximum). Otherwise, allocation is a minimizer.
- Generally, if the solution corresponding to the preceding case ($G_2 = G_0$; $G_1 > G_0$) does not exist, this solution ($G_2 = G_1 = G_0$) is a maximizer.
Summary

- On the maximization of the network weighted throughput in a 2-terminal interference-limited single-cell CDMA:
  - It is always optimal for the important user to transmit at the highest feasible data rate. It may or may not be optimal for the other user to operate at this rate.
  - When \((G_0)^2/\beta\) is “small”, only the important user must operate at “full speed”. This user's optimal SIR is determined by solving an equation of the form \(x^2f'(x)=K(G_0)^2/\beta\). This optimal SIR immediately determines the optimal power-ratios.
  - The other terminal's data rate is determined so that its SIR (product of its processing gain by its power ratio) equals a channel-determined constant.
  - If maximum permitted bit rate is low enough \((G_0)\) is large enough, it becomes optimal to allow both users to transmit at this fastest rate. Optimal power ratios are then determined by solving certain channel-determined equation.

Spread Gain: \(G_i= \frac{R_C}{R_i} \) (Chip_rate / bit_rate) ; \( G_0 = \frac{R_C}{R_{\text{MAX}}} \)
\( \gamma_0 \) solves \(xf'(x)=f(x) \) ; \( \gamma_{00} \) solves \(x^2f'(x)=K(G_0)^2/\beta\) ; \( \beta \) : priority
Spread Gain: \( G_i = \frac{R_C}{R_i} \) (Chip_rate / bit_rate) ; \( G_0 = \frac{R_C}{R_{\text{MAX}}} \)

\( \gamma_0 \) solves \( x f'(x) = f(x) \) ; \( \gamma_{00} \) solves \( x^2 f'(x) = \frac{K(G_0)^2}{\beta} \) ; \( \beta \) : priority

Discussion

- On the maximization of the network weighted throughput in a 2-terminal interference-limited single-cell CDMA:
  - Analysis identifies 3 allocations possibly satisfying some optimality criterion: a BALANCED (‘fair’) allocation, an ‘UNFAIR’ allocation, and a “GREEDY” allocation.
  - The balanced allocation is always sub-optimal: ‘fairness’ is expensive!
  - It is always optimal for the favorite terminal to operate at maximum bit rate.
  - When \( G_0/\sqrt{\beta} \) is larger than a threshold determined by the physical layer through \( f \), both terminals should be admitted at the maximum permissible data rate.
  - The (data) “speed limit” under which the greedy allocation is optimal Decreases, as the favorite terminal grows in importance.
  - If \( G_0 \) is small enough, the greedy allocation actually MINIMIZES the weighted throughput.
Continuing/future work

- Imposing QoS constraints (minimum throughput per terminal)
- Exploring the ‘fairness’ issue (saddle point)
- Considering fixed but dissimilar data rates (spread gains)
- Considering noise
- Mobility (location) issues
- Multiple cells
- Extension to “n” terminals

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$\gamma_0$ solves $xf'(x)=f(x)$; $\gamma_{00}$ solves $x^2f''(x)=\frac{K(G_0)^2}{\beta}$; $\beta$ : priority
Related work

- A paper describing the technical details of maximizing $S(x)/x$ with $S$ a general S-curve is available.
- Another work discussed a “robust” generalized QoS measure for wireless data, and a “game” (decentralized algorithm) in which each terminal chooses power to maximize its own QoS service.
- See wireless.poly.edu

Spread Gain: $G_i = \frac{R_C}{R_i}$ (Chip_rate / bit_rate) ; $G_0 = \frac{R_C}{R_{\text{MAX}}}$

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