Artificial Channel Aided LMMSE Estimation for Time-Frequency Selective Channels in OFDM Context

V. Savaux*,a,b,1, Y. Louët, M. Djoko-Kouam, A. Skrzypczak

aECAM Rennes-Louis de Broglie, CS 29128, 35091 Rennes Cedex 9, France
bIETR-Supélec, Avenue de la Boulaie, CS 47601, 35576 Cesson-Sévigné, France
CZodiac Data Systems, 2 rue de Caen, 14740 Bretteville l’Orgueilleuse, France

Abstract

This paper proposes a linear minimum mean square error-based (LMMSE) channel estimation method, which allows avoiding the necessary knowledge of the channel covariance matrix or its estimation. To do so, a perfectly tunable filter acting like an artificial channel is added at the receiver side. We show that an LMMSE estimation of the sum of this artificial channel and the physical channel only needs the covariance matrix of the artificial channel, and the channel estimation is finally obtained by subtracting the frequency coefficients of the added filter. We call this method artificial channel aided-LMMSE (ACA-LMMSE). Theoretical developments and simulations prove that its performance is close to theoretical LMMSE, and we show that this method reduces the computational complexity, compared to usual LMMSE, due to the covariance matrix used for ACA-LMMSE is computed only once throughout the transmission duration. We put the conditions on the artificial channel parameters to get the expected mask effect. Simulations display the performance of the proposed method, in terms of MMSE and bit error rate (BER). Indeed, the difference of BER between our method and the theoretical LMMSE is less than 2 dB.

*Corresponding author

Email addresses: vincent.savaux@ecam-rennes.com (V. Savaux),
Yves.Louet@supelec.fr (Y. Louët), noise.djoko-kouam@ecam-rennes.fr (M. Djoko-Kouam), Alexandre.Skrzypczak@zodiacaerospace.com (A. Skrzypczak)

1Phone: +33 299058493; Fax: +33 299058419.
1. Introduction

Due to its robustness against multipath channels, orthogonal frequency division multiplexing (OFDM) is used in many telecommunication standards such as digital video broadcasting-terrestrial (DVB-T), long term evolution (LTE), worldwide interoperability for microwave access (WiMAX) etc. Indeed, the use of a guard interval (cyclic prefix (CP)) mitigates the intersymbol interferences (ISI). Moreover, the decomposition of the signal into several narrow frequency bands allows to consider the frequency selective channels as multiple flat-fading subchannels. Thus, a simple one tap per carrier equalization can be done. An efficient channel estimation is then required to make the equalization effective.

In most of transmission designs, some subcarriers called pilots are used to perform the channel estimation. In this case, the position in the time-frequency lattice, the gain and the phase of the pilots are known at both transmitter and receiver sides. The number of pilots and their distribution, given by the standard, depend on the severity of the time-frequency variations of the channel.

The context of this article is the broadcast of an OFDM signal over a time and frequency selective channel.

A wide range of channel estimation techniques using pilot tones exists in literature, as described in [1, 2, 3]. More specifically, the least square (LS) estimation [3] is a low complexity method based on the ratio between the received signal and the pilot value. It offers an acceptable performance compared to the complexity computation. However, this technique is very sensitive to the noise and the possible interpolations along the time or frequency axis in the case of pilots scattered in the OFDM frame. The scaled LS estimation (SLS) described in [3] and [4] improves the performance of the classical LS, but requires the channel
gain and the noise power knowledge for estimation. The maximum likelihood (ML) [5] estimation allows both channel estimation and signal detection, but requires a prohibitive computation complexity. Algorithms reducing its complexity are given in [6] and [7], such as the usual expectation maximisation (EM) algorithm [8] or in [9] in the case of unknown interference.

Based on the minimum mean square error (MMSE) criterion, the algorithm presented in [10] called "two-dimensions Wiener filtering", uses the channel correlation in both time and frequency domains to estimate the channel coefficients all over the time-frequency lattice. Such an estimation is generally too complex to be implemented in practice and can be split into two one-dimension filters. The linear MMSE (LMMSE) estimation described in [11] only requires the frequency covariance matrix which reduces the complexity and offers a performance that is close to the perfect estimation bound. However, the main drawback of LMMSE remains its complexity. Indeed, this method requires the channel covariance matrix and the noise variance knowledge or their estimations. These estimations must be regularly updated when the channel statistics change. Furthermore, a covariance matrix inversion is needed, which implies a number of calculations of order $O(M^3)$ for a $M \times M$ sized matrix. An LMMSE low-rank estimator is presented in [12], having the same performance as classical LMMSE for a lower complexity. In [13], a fast LMMSE channel estimation is given. The covariance matrix and the noise variance are estimated, and a fast algorithm of matrix inversion is used. Despite the complexity improvement of these algorithms, the covariance matrix must be regularly updated to ensure an acceptable level of performance.

In this article, we propose an LMMSE-based channel estimation method which does not necessitate the channel covariance matrix nor its estimation. Indeed, at the receiver side, we add to the received signal an artificial one only
composed of the pilots emitted through a dedicated filter whose parameters are adjustable by the user. In fact, this filter plays the role of an artificial channel. From the estimator-block point of view, the resulting signal is the sum of the OFDM signal through the physical channel and a pilot signal through an artificial channel. The injected pilots have the same scheme as the pilots in the OFDM signal. By choosing well-fitted parameters for the added filter, this artificial channel has the ability of masking the real channel frequency response. The hybrid channel (composed by the sum of the physical and the artificial channels) can then be estimated by the LMMSE method using the covariance matrix of the artificial channel. We get an estimation of the physical channel by subtracting the filter from the estimated hybrid channel. The performance of our proposed estimation is very close to that of theoretical LMMSE. A brief presentation of this method called artificial channel aided-LMMSE (ACA-LMMSE) has already been developed in [14]. The main advantage is that the covariance matrix of the physical channel does not need to be a priori known, nor estimated. Furthermore, an appropriate choice of the parameters of the artificial channel allows to make an LMMSE estimation from the hybrid channel independently from the variations of the channel statistics. Therefore, the covariance matrix of the hybrid channel needs to be computed only once at the beginning of the transmission, while the usual method as presented in [13] needs to track the variations of the channel statistics and its estimated covariance matrix must be updated every OFDM symbol.

This article is organized as follows: Section 2 presents the system model, LS and LMMSE estimation methods. Section 3 details the principle of the proposed estimation method and Section 4 gives the conditions on the parameters to make the method efficient. Simulations results are depicted in Section 5. Finally, we draw our conclusions in Section 6.
2. Background

2.1. Notation

In the rest of the paper, the bold font \( \mathbf{x} \) is used for vectors, the underlined bold font \( \underline{x} \) for matrices and normal font \( x \) for scalar elements. Furthermore, small letters \( x \) are used for signal in time domain and capital letters \( X \) for signal in frequency domain. The mathematical font \( \mathcal{X} \) is used to describe the filters.

2.2. Baseband Transmission Model

Let us consider the transmission of an OFDM signal through a time-varying multipath fading channel following the Wide Sense Stationary Uncorrelated Scattering (WSSUS) model, described in [15]. The discrete-time impulse response of the channel is given by

\[
h(t_n, \tau) = \sum_{l=0}^{L-1} h_l(t_n) \delta(\tau - \tau_l),
\]

where \( L \) is the length of the channel and \( h_l(t_n) \) are independent zero-mean complex Gaussian processes. We assume that the path gains \( h_l(t_n) \) are constant during an OFDM symbol of length \( T_0 \), so we define \( t_n = nT_0 \), where \( n \) is the time index of the considered OFDM symbol. To simplify the notations, we will further note \( h_n(\tau) = h(t_n, \tau) \) and \( h_l(t_n) = h_l,n \). \( \tau_l \) is the path delay defined by \( \tau_l = \beta_l \tau_0 \), where \( \tau_0 \) is the sampling time and \( \beta_l \) is a random real value uniformly and independently distributed in \([0, \beta_{max}]\) interval. Defining the length \( T_{CP} \) of the cyclic prefix (CP), we assume \( \beta_{max} \tau_0 \leq T_{CP} \), i.e. we consider no intersymbol interferences (ISI). We then rewrite (1) as

\[
h_n(\tau) = \sum_{l=0}^{L-1} h_{l,n} \delta(\tau - \beta_l \tau_0).
\]

The corresponding frequency response is obtained by applying a M-points discrete Fourier transform (DFT) on (2). Thus, for all \( m \in \{0, 1, ..., M - 1\} \), we
obtain the following expression for the frequency response of the channel at the frequencies $f_m = \frac{m}{M\tau_0}$:

$$H_{m,n} = \sum_{l=0}^{L-1} h_{l,n} \exp(-2j\pi f_m \tau_l)$$

$$= \sum_{l=0}^{L-1} h_{l,n} \exp(-2j\pi \frac{m}{M} \beta_l).$$  (3)

After removal of the CP and applying the DFT, the discrete transmission equation is given in its vectorial form by (4):

$$U_n = C_n H_n + W_n,$$  (4)

where the vectors $U_n = [U_{0,n}, ..., U_{M-1,n}]^T$ and $W_n = [W_{0,n}, ..., W_{M-1,n}]^T$ are the $M \times 1$ complex vectors of the received signal and the white Gaussian noise in the $n^{th}$ time-slot respectively. $H_n = [H_{0,n}, ..., H_{M-1,n}]^T$ is the channel vector, whose elements are defined in (3). $C_n$ is the $M \times M$ diagonal matrix of the transmitted signal containing on its diagonal the vector $[C_{0,n}, ..., C_{M-1,n}]$. $C_{m,n}$ are data elements from a given constellation or pilots.

2.3. Estimation Methods

We review in this section the principle of the two classical estimation methods, least square (LS) and linear minimum mean square error (LMMSE). In order to develop the theoretical performance of these estimators, given by the Minimum Mean Square Error (MMSE) of the estimation, we here consider the transmission of a training symbol $C_{n_0}$ composed of pilots only. We also suppose that the pilots verify $C_{m,n_0} C_{m,n_0}^* = P$, where $(\cdot)^*$ is the conjugate transformation, and $P$ the pilot signal power.
2.3.1. LS Estimation

From (4), the LS channel estimation is simply done by inverting the diagonal training matrix $\mathbf{C}_n^n$:

$$
\mathbf{H}_{n_0}^{LS} = \mathbf{C}_n^{-1} \mathbf{U}_{n_0} = \mathbf{H}_{n_0} + \mathbf{C}_n^{-1} \mathbf{W}_{n_0}.
$$  (5)

The estimated vector $\mathbf{H}_{n_0}^{LS}$ is composed of elements $H_{m,n_0}^{LS} = H_{m,n_0} + W_{m,n_0}$.

This estimation is very sensitive to the noise. The MMSE of the LS estimation is defined as the minimization of the cost function $J_{LS} = E \{ ||\mathbf{H}_{n_0} - H_{n_0}^{LS}||_F^2 \}$, where $E \{ \} $ is the statistical expectation and $||.||_F$ is the Frobenius matrix norm defined by $||\mathbf{A}||_F = \sqrt{tr (\mathbf{A}^H \mathbf{A})}$, where $tr(.)$ denotes the trace of a matrix. From (5), we get a simpler form of the cost function:

$$
J_{LS} = E \{ ||\mathbf{H}_{n_0} - \mathbf{H}_{n_0}^{LS}||_F^2 \} = E \{ ||\mathbf{W}_{n_0} \mathbf{C}_n^{-1}||_F^2 \}.  \tag{6}
$$

We note $\sigma^2$ the noise variance and $\mathcal{P}$ the pilot signal power, defined by $\sigma^2 = \frac{1}{\mathcal{P}} E \{ ||\mathbf{W}_{n_0}||_F^2 \}$ and $\mathcal{P} = \frac{1}{\mathcal{P}} E \{ ||\mathbf{C}_n||_F^2 \}$ (due to $C_m,n_0 C_{m,n_0}^* = \mathcal{P}$) respectively. Following the complete developments of [3] and the matrix form [16], we obtain the MMSE of the LS estimation:

$$
MMSE_{LS} = \frac{\sigma^2}{\mathcal{P}}.  \tag{7}
$$

The error defined by (7) may be assimilated as a Signal to Noise Ratio (SNR), which confirms the dependence of the LS estimation upon noise. On the other hand, the MMSE does not depend on the channel.
2.3.2. LMMSE Estimation

As developed in [11], the LMMSE estimation exploits the frequency correlation of the channel to filter the previously estimated vector \( \hat{H}_{no} \):

\[
\hat{H}_{n}^{LMMSE} = R_H (R_H + \sigma^2 \begin{pmatrix} C_{no} & C_{no}^H \end{pmatrix}^{-1})^{-1} \hat{H}_{no}^{LS}
\]

\[
= R_H (R_H + \frac{\sigma^2}{P} I)^{-1} \hat{H}_{no}^{LS},
\]

(8)

where \( R_H \) is the \( M \times M \) frequency covariance matrix defined by \( R_H = E \left\{ H_{no} H_{no}^H \right\} \), with \( (\cdot)^H \) the Hermitian transpose and \( I \) the identity matrix. We detail in Section 2.4 the development of this matrix. The MMSE of the LMMSE estimation is obtained by the minimization of the following cost function:

\[
J_{LMMSE} = E \left\{ ||H_{no} - \hat{H}_{no}^{LMMSE}||_F^2 \right\}
\]

\[
= tr \left( (R_H^{-1} + \frac{\sigma^2}{P} I)^{-1} \right),
\]

(9)

Following the same developments of [3] and the matrix form [16] as for LS case, we obtain the MMSE of the LMMSE estimation:

\[
MMSE_{LMMSE} = \frac{1}{P/\sigma^2 + tr \left\{ R_H^{-1} \right\}}.
\]

(10)

As \( R_H \) is an Hermitian matrix, definite and positive matrix [4], its eigenvalues are positive, what ensure \( tr (R_H^{-1}) > 0 \). Comparing the MMSE of LS (7) and LMMSE (10), we get

\[
MMSE_{LS} > MMSE_{LMMSE}, \ \forall P/\sigma^2 > 0.
\]

The estimation error of LMMSE is less than that of LS. It confirms that the LMMSE estimation is more efficient than the LS estimation.

In the case of scattered pilots in the frame, an interpolation is needed. For
the LS estimation, linear or polynomial interpolations can be used in order to estimate the channel over the complete time-frequency lattice. In [12], it is shown that the LMMSE filter in scattered pilots case directly plays the role of interpolator and has a similar MMSE to the one in the preamble case. Due to these interpolations, it is impossible to develop the analytical MMSE expressions. However, the comparison of MMSE between LS and LMMSE in the case of scattered pilots in the frame will be shown by simulations in Section 5.

2.4. Expression of the Frequency Covariance Matrix $\mathbf{R}_H$

The frequency covariance matrix used in (8) is defined by $\mathbf{R}_H = E \left\{ \mathbf{H}_{n_0} \mathbf{H}_{n_0}^H \right\}$. By noting $(u, v)$ the indexes of the rows and columns of this matrix, we get the general term $(\mathbf{R}_H)(u, v)$ from [12] and [17]:

$$
(\mathbf{R}_H)(u, v) = E \left\{ H_{u,n_0} H_{v,n_0}^* \right\} = \sum_{l=0}^{L-1} \int_0^{\beta_{max}} \Gamma(\beta_l) e^{-2j\pi \frac{u-v}{\tau_{max}}} \beta_l d\beta_l,
$$

where $\Gamma(\beta_l)$ is the multipath intensity profile and $\beta_{max}$ verifies $\tau_{max} = \beta_{max} \tau_0$, with $\tau_{max}$ the maximum path delay.

In practice, the parameters $\Gamma(\beta_l)$, $L$ and $\beta_{max}$ are a priori unknown at the receiver side, which makes this formulation of covariance matrix unusable. Furthermore, these parameters may change during the transmission. It would also make the use of the covariance matrix inadequate, or require an update. A solution is to estimate the matrix $\mathbf{R}_H$ with the help of the LS estimation of the channel. We denote $\tilde{\mathbf{R}}_H$ the estimated channel covariance matrix, such as

$$
\tilde{\mathbf{R}}_H = \mathbf{H}_{n_0}^{LS} \mathbf{H}_{n_0}^{LS^H}.
$$

In the following, when LMMSE is performed with the covariance matrix (12),
we will refer to it as the "usual case". As we here consider the case of a fast-varying channel, the channel covariance estimation in (12) has to be performed on each OFDM symbol. As a direct consequence, this considerably increases the complexity of the LMMSE estimation in (8). More details concerning the complexity are given Section 3.3. Other approximations of the covariance matrix can be performed, as presented in [13], but that requires regular updates in order to track the channel variations too.

3. LMMSE Estimation when the frequency channel covariance matrix \( R_H \) is unknown

3.1. Principle of the Method

In the previous section, we discussed the difficulty to design the covariance matrix \( R_H \). In most cases, the receiver does not know the statistical parameters of the channel. In the case of estimation of this matrix, updates must be regularly performed in order to track the channel variations. We here propose an efficient method close to theoretical LMMSE estimation, but without any need of the channel parameters nor the estimation of the covariance matrix. The method is illustrated in Fig. 1. An artificial signal (only composed of pilots) is distorted by a filter \( \mathcal{G} \) playing the role of an artificial channel, and added to the received signal. The pilots of the artificial signal have the same position in the time-frequency lattice as the pilots of the OFDM received signal. The position tallying with the data carriers in the OFDM signal are replaced by zeros in the artificial signal. We assume that the artificial signal is perfectly synchronized with the received signal. The time-varying coefficients of the filter \( \mathcal{G} \) follow a statistic perfectly known and tunable by the receiver. This statistic is chosen in order to match a WSSUS channel. The coefficients of the filter \( \mathcal{G} \) are randomly varying like an artificial channel. Consequently, we use a channel terminology
Figure 1: Diagram of ACA-LMMSE estimation in a simplified transmission-reception chain.
to describe $\mathcal{G}$.

From the estimator-block point of view, the pilots are distorted by the sum of the physical channel and the artificial channel. The resulting channel, noted $\mathcal{K} = \mathcal{H} + \mathcal{G}$, is called hybrid channel in the following discussions. We have any knowledge of the statistical properties of the physical channel nor of their evolution. We then propose a way to design $\mathcal{G}$ such that the covariance matrix $\mathbf{R}_K$ used in the LMMSE estimation of the hybrid channel can be entirely computed using the parameters of $\mathcal{G}$. In this way, we will justify the covariance matrix approximation $\mathbf{R}_K \approx \mathbf{R}_G$. In other words, $\mathcal{G}$ plays the role of "mask" for $\mathcal{H}$.

As $\mathcal{G}$ is perfectly known, we retrieve the estimation of $\mathcal{H}$ by subtracting $\mathcal{G}$ from the LMMSE estimation of the hybrid channel $\mathcal{K}$.

As we make an LMMSE-based estimation of the physical channel with the help of an artificial channel $\mathcal{G}$, we refer the proposed method as artificial channel aided-LMMSE (ACA-LMMSE) in the rest of the article.

### 3.2. ACA-LMMSE Estimation

In this section we develop the mathematical expression of the proposed estimation method. We then deduce the theoretical performance of ACA-LMMSE in term of MMSE. To do so, we here consider the transmission of a training symbol (named preamble) $\mathbf{C}_{m,n_0}$ composed only of pilots $C_{m,n_0}$, with $C_{m,n_0} \neq 0$.

We note $\mathbf{S}_{n_0}$ the received pilot signal from the estimator-block point of view. The expression of $\mathbf{S}_{n_0}$ is given by

$$
\mathbf{S}_{n_0} = \mathbf{K}_{n_0} \mathbf{C}_{n_0} + \mathbf{W}_{n_0}.
$$

By analogy with (2), the impulse response of the artificial channel $\mathcal{G}$ is given
by

\[ g_n(\tau) = \sum_{d=0}^{D-1} g_{d,n} \delta(\tau - \beta_d \tau_0), \]  

where \( D \) is the number of paths of the artificial channel \( \mathcal{G} \) and \( g_{d,n} \) its path gains for \( d = 0, 1, ..., D - 1 \).

Remembering (8), at this step of the transmission chain, the LMMSE of the hybrid channel is performed as

\[
\hat{K}_{n_0}^{LMMSE} = R_K \left( R_K + \sigma^2 \left( C_{n_0} C_{n_0}^H \right)^{-1} \right)^{-1} \hat{K}_{n_0}^{LS} \]

\[ = R_K \left( R_K + \frac{\sigma^2}{P} I \right)^{-1} \hat{K}_{n_0}^{LS}, \]  

where \( R_K \) is the \( M \times M \) complex covariance matrix of \( \mathcal{K} \). Suppose that the noise variance is known or estimated with an adapted algorithm for OFDM transmissions [18], [19], [20]. The vector \( \hat{K}_{n_0}^{LS} \) contains the LS estimation of the hybrid channel coefficients on the preamble. In order to do the LMMSE estimation of the hybrid channel in (15), the covariance matrix \( R_K = R_{H+G} \) has to be calculated. However \( R_K \) is unknown, as the statistics of \( \mathcal{H} \) are unknown. Our solution aims at setting the statistics of \( \mathcal{G} \) (recalling that these statistics are fixed by the user) allowing the approximation \( R_K \approx R_{\mathcal{G}} \). Justifying this condition, we ensure that the LMMSE estimation of \( \mathcal{K} \) can be performed irrespective of the variation of \( \mathcal{H} \), and then \( R_{\mathcal{G}} \) needs to be computed only once. Section 4 explains how to choose the parameters of \( \mathcal{G} \) to ensure \( R_K \approx R_{\mathcal{G}} \). From (15), we get an estimation of the physical channel noted \( \hat{H}_{n_0}^{ACA-LMMSE} \):

\[
\hat{H}_{n_0}^{ACA-LMMSE} = \hat{K}_{n_0}^{LMMSE} - G_{n_0}. \]  

Although the coefficients of \( \mathcal{G} \) are randomly generated (by a known statistic),
they can be stored in a memory after their generation so that $\mathbf{G}_{no}$ can be completely accessible for the estimation step of (16).

The MMSE of the ACA-LMMSE is defined as the minimization of the following cost function $J_{ACA-LMMSE}$:

$$J_{ACA-LMMSE} = E \left\{ \left\| \mathbf{H}_{no} - \hat{\mathbf{H}}_{no}^{ACA-LMMSE} \right\|^2_F \right\}$$

$$= E \left\{ \left\| \mathbf{H}_{no} - (\hat{\mathbf{K}}_{no}^{LMMSE} - \mathbf{G}_{no}) \right\|^2_F \right\}$$

$$= E \left\{ \left\| \mathbf{H}_{no} + \mathbf{G}_{no} - \hat{\mathbf{K}}_{no}^{LMMSE} \right\|^2_F \right\}. \quad (17)$$

Remembering that $\mathbf{K}_{no} = \mathbf{H}_{no} + \mathbf{G}_{no}$ from (13), we obtain:

$$J_{ACA-LMMSE} = E \left\{ \left\| \mathbf{K}_{no} - \hat{\mathbf{K}}_{no}^{LMMSE} \right\|^2_F \right\}. \quad (18)$$

The analogy between (18) and the cost function of LMMSE given by (9) directly leads to the MMSE of the ACA-LMMSE estimation, using (10):

$$MMSE_{ACA-LMMSE} = \frac{1}{P/\sigma^2 + tr(\mathbf{R}_K^{-1})}. \quad (19)$$

We observe that MMSE of ACA-LMMSE (19) has the same formulation as the theoretical LMMSE. It then theoretically proves the efficiency of ACA-LMMSE. However, since $\mathbf{K}$ is unknown, if we verify the approximation $\mathbf{R}_K \approx \mathbf{R}_G$, (19) proves that the performance of ACA-LMMSE is driven by an appropriate choice of $\mathbf{G}$. As LMMSE plays the role of an interpolator, our solution is also valid in the case of a scattered pilot distribution. Although the closed form of MMSE is impossible to derive in this case, we will compare our solution with LMMSE and LS by simulations in Section 5.
3.3. Complexity comparison

We now compare the complexity of the proposed ACA-LMMSE with the usual LMMSE performed with the estimated channel covariance matrix of (12). We denote $M \times M$ the dimension of the matrices $\tilde{R}_H$ and $R_H$. As summarized in Table 1, three operations are taken into account: the matrix multiplication and inversion made in LMMSE estimation (8), the operations performed to create the covariance matrix and the number of updates required due to the time-varying channel.

In both cases, performing $R_H(R_H + \frac{\sigma^2}{\gamma}I)^{-1}$ requires a complexity equal to $\mathcal{O}(M^6)$, i.e. $\mathcal{O}(M^3)$ for the matrix inversion and $\mathcal{O}(M^3)$ for the multiplication. Obviously, $R_H$ is replaced by its estimate $\tilde{R}_H$ in usual LMMSE case. The second operation is the generation of the covariance matrix. In the usual LMMSE case, $\tilde{R}_H = \hat{H}_n^L S \hat{H}_n^L$ has a complexity equal to $\mathcal{O}(M^2)$. In ACA-LMMSE case, $R_H$ is known in advance at the receiver (e.g. it can be stored in a memory), so the user does not have to update it. In Table 1, $N$ is the number of OFDM symbols for a given transmission. Since we consider a fast time-varying channel (i.e. its gain changes between two consecutive OFDM symbols), the covariance matrix $R_H$ must be estimated every symbol in the usual LMMSE case. As a consequence, $R_H(R_H + \frac{\sigma^2}{\gamma}I)^{-1}$ must also be updated $N$ times in the usual LMMSE case. In ACA-LMMSE case, $R_H$ is fixed and does not require any update. Finally, the complexity of usual LMMSE is $N$ times higher than the one of ACA-LMMSE for the matrix multiplication and inversion. Furthermore, usual LMMSE requires a complexity equal to $\mathcal{O}(NM^2)$ in order to update the covariance matrix, while ACA-LMMSE does not require any update. In the context of fast varying channel, ACA-LMMSE is then globally $N$ times less complex than usual LMMSE.
4. Choice of the Artificial Channel Parameters

The aim of this section is to give the expression of $\mathbf{R}_K$ on one hand, and to perform the adapted choice of the parameters allowing to justify the approximation $\mathbf{R}_K \approx \mathbf{R}_G$ on the other hand. To illustrate this approximation, we use the practical \textit{US Consortium} channel model coming from the Digital Radio Mondiale (DRM) standard [21] designed for the digital audio broadcasting over the currently AM frequency bands. Although we use a practical model from DRM standard in this section, we will extend the method for any WSSUS model in Section 5. \textit{US Consortium} is a 4-paths channel (i.e. $L = 4$) with a maximum delay $\tau_{\text{max}} = 2.2$ ms and a decreasing intensity profile. Details concerning \textit{US Consortium} are given in Table 2. The term Gaussian refers to the Doppler spectrum profile [22]. The transmitted OFDM signal is composed of 148 independent carriers, according to robustness Mode C of the standard. Table 3 summarizes the parameters used for the simulations. The assumption $\mathbf{R}_K \approx \mathbf{R}_G$ is then shown with the example of the \textit{US Consortium} channel.

4.1. Expression of the Channel Covariance Matrix

In our mathematical developments, we suppose that the artificial channel follows the WSSUS model. Remembering that $D$ is the number of paths of the artificial channel $\mathcal{G}$ and $g_{d,n}$ its path gains for $d = 0, 1, ..., D - 1$, from (2), we give the impulse response of the hybrid channel $\mathcal{K}$:

\[
k(\tau) = (h + g)(\tau) = \sum_{l=0}^{L-1} h_{l,n} \delta(\tau - \beta_l \tau_0) + \sum_{d=0}^{D-1} g_{d,n} \delta(\tau - \beta_d \tau_0) \\
= \sum_{b=0}^{B-1} \gamma_{b,n} \delta(\tau - \beta_b \tau_0),
\] (20)
where \( B \leq L + D \) is the number of paths of the hybrid channel \( k(\tau) \) and \( \gamma_{b,n} \) its path gains with \( b = 0, 1, ..., B - 1 \). Thus, for a given \( \beta_b \), the gain \( \gamma_{b,n} \) may be equal to \( h_{b,n} \) (if \( g_{b,n} = 0 \)), \( g_{b,n} \) (if \( h_{b,n} = 0 \)) or \( h_{b,n} + g_{b,n} \) if \( h(\tau) \) and \( g(\tau) \) have a common path at \( \tau = \beta_b \tau_0 \). Using (3) and (11), we give the general term \( (R_{K})_{u,v} = E \{ K_{u,n} K_{v,n,*} \} \) of the matrix \( R_K \):

\[
(R_K)_{u,v} = \sum_{b=0}^{B-1} \int_0^{\beta_{max}} \Gamma(\beta_b) e^{-2j\pi \frac{(u-v)}{\beta_b} \beta_b} d\beta_b.
\]

Equation (21) highlights three parameters which have an influence on the covariance matrix \( R_K \): the number of paths \( B \) of the hybrid channel, with \( B \leq D + L \), the maximum delay \( \tau_{max} = \beta_{max} \tau_0 \), with \( \tau_{max} = \max(\tau_{max}^{(G)},\tau_{max}^{(H)}) \) and the multipath delay profile \( \Gamma(\beta_b) \).

The physical channel \( \mathcal{H} \) being unknown, the exact statistical parameters of \( K \) are unknown. Thus, we can not use the covariance matrix \( R_K \) described by (21). We then perform the LMMSE estimation of the hybrid channel using the covariance matrix of \( \mathcal{G} \), whose parameters are perfectly known and controllable, and defined by :

\[
(R_G)_{u,v} = \sum_{d=0}^{D-1} \int_0^{\beta_{max}^{(G)}} \Gamma(\beta_d) e^{-2j\pi \frac{(u-v)}{\beta_d} \beta_d} d\beta_d.
\]

The next subsections aim to characterize the three parameters in order to get the approximation \( R_K \approx R_G \), i.e. make (22) as close as possible to (21).

### 4.2. Discussion on the Choice of the Parameters

In this subsection, we focus on the characterization of the parameters \( \tau_{max}^{(G)} \), \( D \) and \( \Gamma(\beta_b) \). To do so, firstly, our choice is driven by using some basic features about OFDM. Simulations will then confirm our choices in subsections 4.3, 4.4, and 4.5 secondly.

As channel \( \mathcal{H} \) is unknown, \( \tau_{max}^{(H)} \) is consequently unknown. On the other
hand, we suppose that the system is designed in order to verify $T_{CP} \geq \tau^{(H)}_{\max}$, i.e. there is no ISI. In order to make equal the bounds in the integrals of (21) and (22), $\beta_{\max}$ in (21) must be chosen equal to $\beta^{(G)}_{\max}$. To ensure that $\beta_{\max} = \beta^{(G)}_{\max} \geq \beta^{(H)}_{\max}$, we then choose $\beta^{(G)}_{\max} = T_{CP}$.

The goal of the artificial channel is to mask the physical one. As a consequence, the number of artificial paths $D$ must be chosen large in comparison to $L$. However, as $L$ is unknown, we must fix an arbitrary large $D$ value. Due to the discrete time formulation, the length of the impulse response (limited by $\tau^{(G)}_{\max}$) of the filter is finite. We can then fix an upper limit to the number $D$ equal to $\tau^{(G)}_{\max}/\tau_0$. The effect of the number of artificial paths is depicted further in Subsection 4.4.

The multipath intensity profile $\Gamma(\beta_0)$ can be a priori chosen in an infinite set of functions. Nevertheless, the choice is only limited by a practical consideration: expression (22) must be integrable. However, the decreasing exponential shape is commonly used ([12], [23] and [24]) in a large number of channel models, so we will use this shape. Nevertheless, in Subsection 4.5 we show that other shapes can also be used in (22).

4.3. Discussion on the Choice of the Maximum Delay $\tau^{(G)}_{\max}$

According to (21), we verify the validity of ACA-LMMSE for any value of the maximum delay $\beta_{\max} = \beta^{(G)}_{\max} \geq \beta^{(H)}_{\max}$, with $\beta^{(G)}_{\max} \in [\beta^{(H)}_{\max}, T_{CP}]$. Since $\beta^{(H)}_{\max}$ is unknown, we show that $\beta^{(G)}_{\max}$ can be chosen equal to $T_{CP}$ without degradation of performance. Figure 2 depicts the three-dimension curve of Bit Error Rate (BER) using the ACA-LMMSE estimation of the US Consortium channel as a function of both $\beta^{(G)}_{\max}/\beta^{(H)}_{\max}$ and $E_b/N_0$. According to the aforementioned recommendations, the number of paths $D$ is set to 20 paths and $\Gamma(\beta_0)$ follows a decreasing exponential profile.

We observe two zones in the BER curve. For $\beta^{(G)}_{\max}/\beta^{(H)}_{\max} \leq 1$, the error
floor of the BER is larger than 0.1, for any considered value of $E_b/N_0$, and for $\frac{\beta_{\max}^{(G)}}{\beta_{\max}^{(H)}} \geq 1$, the BER does not reach any error floor in the considered $E_b/N_0$ values, according to an efficient channel estimation. The error floor of the zone $\frac{\beta_{\max}^{(G)}}{\beta_{\max}^{(H)}} \leq 1$ is due to a bad LMMSE hybrid channel estimation. Indeed, LMMSE method requires a correctly sized covariance matrix, i.e. the rank of the covariance matrix must be equal to the length of the channel path delay, according to [12]. $\mathcal{H}$ being unknown, we use $\mathbf{R}_G$ to perform the estimation, and the rank of $\mathbf{R}_G$ is equal to the length of the artificial channel $\mathcal{G}$. In the case $\frac{\beta_{\max}^{(G)}}{\beta_{\max}^{(H)}} \leq 1$, the length of the hybrid channel $\mathcal{K} = \mathcal{H} + \mathcal{G}$ is equal to that of the physical channel $\mathcal{H}$, so the rank of the covariance matrix $\mathbf{R}_G$ is less than the length of the hybrid channel. In these conditions, the LMMSE method does not perform an efficient hybrid channel estimation, and so the channel $\mathcal{H}$ can not be efficiently estimated. In the case $\frac{\beta_{\max}^{(G)}}{\beta_{\max}^{(H)}} \geq 1$, the condition on the rank of the covariance matrix $\mathbf{R}_G$ equal to the length of the hybrid channel is respected. Indeed, the length of the hybrid channel $\mathcal{K} = \mathcal{H} + \mathcal{G}$ is equal to that of the artificial channel $\mathcal{G}$. $\mathcal{K}$ is then well estimated with LMMSE method and the channel estimation is efficiently performed with ACA-LMMSE. We also observe that for $\beta_{\max}^{(G)} \in \left[\beta_{\max}^{(H)}, T_{CP}\right]$, the BER reaches the same value for a given $E_b/N_0$. The delay parameter $\beta_{\max}^{(G)}$ can then be chosen equal to $T_{CP}$ without degradation of performance. It justifies the first a priori choice $\beta_{\max}^{(G)} = T_{CP}$.

4.4. Discussion on the Choice of the Number of paths $D$ of the Artificial Channel

In this subsection, we study the effect of the number of paths $D$ of the artificial channel, the parameters $\tau_{\max}$ and $\Gamma(\beta_0)$ being chosen according to the previous recommendations. By hypothesis, the number of paths $L$ of the physical channel is unknown, thus, $B$ is also unknown in (21). We then use the covariance matrix defined by (22) to perform the LMMSE estimation of the hybrid channel $\mathcal{K}$. Recalling that $B \leq L + D$, the larger $D$ compared to $L$, the
less the error between $D$ and $B$. The physical consequence is that the gain of
the channel becomes negligible in comparison to that of the added filter, which
justifies the masking effect. We remind that $L = 4$ in our example.

On one hand, we know that the channel is defined by the non-null eigenvalues
of its covariance matrix [25]. On the other hand, as the channel covariance
matrices are Hermitian, comparing eigenvalues of two covariance matrices is
equivalent to comparing the matrices themselves [26]. In order to justify the
approximation $\mathbf{R}_K \approx \mathbf{R}_G$, we compare the eigenvalues of the matrices $\mathbf{R}_K$ and
$\mathbf{R}_G$. For $m = 0, ..., \alpha - 1$ we define $\lambda^{(K)}_m$ and $\lambda^{(G)}_m$ the eigenvalues of $\mathbf{R}_K$ and
$\mathbf{R}_G$ respectively. Figure 3 depicts the normalized error noted $\varepsilon_m$ as a function
of $m$ and defined by

$$\varepsilon_m = \frac{|\lambda^{(K)}_m - \lambda^{(G)}_m|}{\max_{m=0}^{\Bbb{m}}(\lambda^{(G)}_m)}, \tag{23}$$

where $\max_{m=0}^{\Bbb{m}}(\lambda^{(G)}_m)$ is the largest eigenvalue of the matrix $\mathbf{R}_G$. Four curves of
error are considered, tallying with $D = 5, 10, 20$ and 40 paths. The eigenvalues
are stored in the growing order $\lambda^{(G)}_m \leq \lambda^{(G)}_{m+1}$ and $\lambda^{(K)}_m \leq \lambda^{(K)}_{m+1}$.

Two parts are noticeable in Fig. 3. For $m = 0$ to 93, the error is null
and for $m = 94$ to 147, the error is non-null. This is because the channel
gain is contained in the last eigenvalues, tallying with the length of the impulse
response. We observe in Fig. 3 that the larger the number of paths $D$, the lower
the normalized error $\varepsilon_m$. Indeed, for $D = 40$, the error $\varepsilon_m$ is less than 0.05 for
$m = 0, ..., \alpha - 1$. From Fig. 3, we can fix an arbitrary value $D$ equal to 1/3
or 1/2 of the upper limit $\tau^{(G)}_{\text{max}} / \tau_0$. Indeed, for sufficiently large $D$, the matrices
$\mathbf{R}_G$ and $\mathbf{R}_K$ have their eigenvalues almost equal, i.e. for $m = 0, ..., \alpha - 1$ we
have $\lambda^{(G)}_m \approx \lambda^{(K)}_m$. It then justifies the approximation $\mathbf{R}_G \approx \mathbf{R}_K$. 

20
4.5. Discussion on the Choice of the Multipath Intensity Profile $\Gamma(\beta)$ of the Artificial Channel

In this subsection, we study the effects of the multipath intensity profile $\Gamma(\beta)$ on the estimation efficiency. The parameters $\tau_{\text{max}}$ and $D$ are chosen according to the previous recommendations. As already stated, since $K$ is unknown, we approach the function $\Gamma(\beta)$ of the hybrid channel by the one of the artificial channel $\mathcal{G}$, which is chosen by the user. Since a decreasing exponential profile is a usual model for the gain of the multipath channels (such as $US$ Consortium), we a priori suppose that it is the best profile for $\Gamma(\beta)$ to guarantee the masking effect.

Although every integrable function can be used in (22), we will here consider three shapes for the multipath intensity profile: a decreasing exponential profile (d.e.p.) (noted $\Gamma_{\text{dep}}(\beta)$), a constant profile (c.p.) (noted $\Gamma_{\text{cp}}(\beta)$) and a growing exponential profile (g.e.p.) (noted $\Gamma_{\text{gcp}}(\beta)$), and given by (24) (25) and (26) respectively and depicted in Fig. 4.

$$
\Gamma_{\text{dep}}(\beta) = \begin{cases} 
C e^{-\beta/\tau_{\text{max}}} & \text{if } \beta \in [0, \beta_{\text{max}}] \\
0, & \text{else,}
\end{cases} \quad (24)
$$

$$
\Gamma_{\text{cp}}(\beta) = \begin{cases} 
C & \text{if } \beta \in [0, \beta_{\text{max}}] \\
0, & \text{else,}
\end{cases} \quad (25)
$$

$$
\Gamma_{\text{gcp}}(\beta) = \begin{cases} 
C e^{\beta/\tau_{\text{max}}} & \text{if } \beta \in [0, \beta_{\text{max}}] \\
0, & \text{else,}
\end{cases} \quad (26)
$$

where $C$ is a normalization constant. From (22) and with (24) to (26), we obtain the corresponding covariance matrices, whose general terms are given from (27) to (29). The development from (22) to obtain (27) given (24) is performed in
Appendix B. Equations (28) and (29) are derived similarly.

\[
(R_{\mathcal{G}})^{d_{ep}}_{u,v} = DC \cdot \frac{1 - e^{-2 j \pi \frac{(u-v)}{M} \beta_{\text{max}}}}{1 + 2 j \pi \frac{(u-v)}{M} \beta_{\text{max}}},
\]

(27)

\[
(R_{\mathcal{G}})^{c_{ep}}_{u,v} = DC \cdot \frac{1 - e^{-2 j \pi \frac{(u-v)}{2M} \beta_{\text{max}}}}{2 j \pi \frac{(u-v)}{M} \beta_{\text{max}}},
\]

(28)

\[
(R_{\mathcal{G}})^{g_{ep}}_{u,v} = DC \cdot \frac{1 - e^{-2 j \pi \frac{(u-v)}{M} \beta_{\text{max}}}}{1 - 2 j \pi \frac{(u-v)}{M} \beta_{\text{max}}},
\]

(29)

Fig. 5 displays the comparison between the classical LMMSE estimation (with the known $R_{\mathcal{H}}$) and the ACA-LMMSE using the three proposed multipath intensity profiles. To get a perfect control of $R_i$, we fix $\beta = 20$ paths. Simulation shows that our proposed solution is really close to the LMMSE estimation performance and also outperforms the classical LS estimation with interpolation. We observe only a 2 dB loss for our proposed solution with respect to the LMMSE estimator. Furthermore, in ACA-LMMSE case, we do not have any knowledge on the statistics of the physical channel, contrary to the theoretical LMMSE estimator. This 2 dB loss comes from the approximation of the multipath intensity profile of the hybrid channel $\mathcal{K}$ by $\mathcal{G}$. In addition, we can observe that the shape of the intensity profile do not have a significant impact on the BER performance. Indeed, the decreasing exponential profile leads to only slightly better results than the other two profiles. This is due to the shape of the considered channel, which has also a decreasing intensity profile. The filter $\mathcal{G}$ matches then better with the physical channel. Nevertheless, with sufficiently large $D$, $\mathcal{G}$ acts like a mask for $\mathcal{H}$. The ACA-LMMSE method then leads to good estimations for any choice of $\Gamma(\beta)$. This choice is not a necessary condition for the feasibility of the proposed method.
4.6. Rules to apply ACA-LMMSE

This subsection aims at summarizing the way to perform ACA-LMMSE estimation. We consider a practical case, i.e. the physical channel is perfectly unknown. We only assume a transmission without ISI. The three parameters are chosen as follows:

1. $\tau_{\text{max}}^{(G)} = T_{CP}$ ensures $\tau_{\text{max}}^{(G)} \geq \tau_{\text{max}}^{(H)}$, with good efficiency.
2. $D$ is chosen lower than the upper limit $\tau_{\text{max}}^{(G)}/\tau_0$ equal to the number of samples in the impulse response of the artificial channel.
3. $\Gamma(\beta)$ must be chosen in order to make (22) integrable. However, the decreasing exponential model is frequently taken in literature as a good representation of most physical channels ([12], [23] and [24]). $\Gamma_{\text{dep}}(\beta)$ can then be used as a good first approach.

5. Simulations and Discussion

In this section, we use the same simulation setup as presented in Section 4, and the binary signal is mapped with a 16-QAM constellation. The parameter of the used US Consortium channel are summarized in Table 2.

5.1. MMSE of the ACA-LMMSE Estimation

Figs. 6 and 7 compare the performance of our method (ACA-LMMSE) with LS and theoretical LMMSE in terms of MMSE, estimated by the value $E \left\{ \frac{1}{M} \sum_{m=0}^{M-1} |H_m - \hat{H}_m|^2 \right\}$, according to two pilot distributions: a preamble-based (PB, Fig. 6), according to our developments, and a scattered pilot distribution (SP, Fig. 7), according to the DRM standard. Concerning the SP case for LS, a polynomial interpolation over the time and frequency dimensions is made as it efficiently limits the degradation of the MSE. According to robustness C, we place one pilot every 4 carriers along the frequency axis and 1 pilot every
2 carriers along the time axis. The filter $G$ has the following parameters: $D = 15$, $\tau_{\text{max}}^{(G)} = T_{\text{CP}} = 5.33$ ms and the delay profile follows a decreasing exponential profile. This configuration is coherent regarding the previous requirements.

Considering first the PB case, we observe that the appropriate choice of the characteristics of $G$ allows to reach an MMSE of ACA-LMMSE estimation less than 2 dB above the MMSE of theoretical LMMSE. The ACA-LMMSE performance is slightly degraded compared to LMMSE, as our estimation uses the covariance matrix $R_H$ which is naturally different from $R_H$. It matches with the theoretical expressions (10) and (19). Now comparing ACA-LMMSE with LS, it clearly outperforms LS as it offers an MMSE gain around 10 dB. For the SP distribution, we notice that we have exactly the same MMSE evolutions, this proves that our technique is also efficient in this case. Indeed, in this context, the MMSE depends on the nature of the interpolation technique, for which no theoretical expression of the MMSE can be found in the literature. Note then that the differences between the SP and PB cases are due to the interpolation technique that inevitably degrades the estimation quality.

5.2. Comparison of ACA-LMMSE with other methods

Fig. 8 displays the BER of three possible implementations of LMMSE, LS and perfect estimations versus $E_b/N_0$. We compare the performance of the proposed ACA-LMMSE with LMMSE performed with the exact channel covariance matrix $R_H = H_n H_n^H$ on one hand, and with LMMSE performed with the estimated one $\tilde{R}_H$ (12) on the other hand.

We observe that the three LMMSE estimations outperform LS from $E_b/N_0 = 15$ dB, and do not reach an error floor. In the case of LS, this error floor is due to the interpolation between pilots and it is equal to $4.10^{-2}$. According to [12], when performed with $R_H$, LMMSE is optimal and almost reaches the perfect estimation. ACA-LMMSE and usual LMMSE performed with $\tilde{R}_H$ almost reach
the same performance, the gap of $E_b/N_0$ between the two curves being less than 0.2 dB. Furthermore, the error between these estimations and the perfect one is less than 2 dB. In both cases, it is due to the approximation of the covariance matrix: $\hat{R}_H$ instead of $R_H$ in usual LMMSE case, and $R_G \approx R_K$ in ACA-LMMSE case. We conclude that, for an equivalent BER performance, ACA-LMMSE is $N$ times less complex than usual LMMSE.

5.3. Suitability of ACA-LMMSE in general WSSUS Channel Models

In order to show that ACA-LMMSE is suitable to all WSSUS-based models, the physical channels used to simulate Fig. 9 have been randomly simulated, i.e. $L$ and $\tau_{max}^{(H)}$ are random variables and $\Gamma(\beta)$ is a random function. Thus the number of paths $L$ of the physical channel $\mathcal{H}$ follows a uniform distribution between 2 and 14. The maximum delay $\tau_{max}^{(H)}$ also follows a uniform distribution over 1 ms and 4 ms. The BER is averaged using to 30 runs of 70 frames, which is equivalent to $4 \times 10^6$ bits. At each run, all the parameters change.

Fig. 9 compares the BER performance of LMMSE and ACA-LMMSE estimation for these physical random channels. For LMMSE, the parameters at each run are supposed to be known. Parameters of ACA-LMMSE are the same as mentioned previously: $D = 20$ paths, $\tau_{max}^{(G)} = T_{CP} = 5.33$ ms and $\Gamma(\beta)$ follows a decreasing exponential profile.

We observe that the difference between the BER curves of LMMSE and ACA-LMMSE is less than 0.5 dB. Furthermore, the difference of BER between ACA-LMMSE and perfect estimation is about 1 dB. These results match with the ones we obtained with $US$ Consortium channel. It shows that the proposed method can be adapted for all channels based on WSSUS model. It shows the robustness of the ACA-LMMSE method.
5.4. Reduction of Implementation Complexity

In this subsection, we show that the complexity of ACA-LMMSE can be reduced without loss of efficiency. Indeed, due to the random nature of the artificial channel coefficients, it is noted in Section 3 that the coefficients of $\mathcal{G}$ are stored in order to be reused in the filter subtraction step. It raises two problems: the generation of time varying coefficients and their storage. We propose to remove these two difficulties by using a simple filter, whose coefficients are fixed. We can rewrite the impulse response of $\mathcal{G}$ (14) without subscript $n$ as:

$$g(\tau) = \sum_{d=0}^{D-1} g_d \delta(\tau - \beta_d \tau_0).$$

(30)

It can be seen that $g(\tau)$ is invariant from one OFDM symbol to the other. In order to keep the masking effect of $\mathcal{G}$ on $\mathcal{H}$ on one hand, and the same covariance matrix (27) as previously on the other hand, we fix $\tau^{(G)}_{\text{max}} = T_{CP}$, $D = 20$ paths. In order to approach the multipath intensity profile $\Gamma(\tau)$, we impose that the coefficient gain $|g_d|^2$ which is a function of $\tau$ follows a decreasing exponential profile. The $D$ paths are equally spaced on the interval $[0, \tau^{(G)}_{\text{max}}]$, with $g_0$ tallying with $\tau = 0$ and $g_{D-1}$ tallying with $\tau = \tau^{(G)}_{\text{max}} = T_{CP}$. The phase $\phi(g_d)$ of each path coefficient is randomly chosen, following a uniformly distribution on $[0, 2\pi]$.

Fig. 10 compares the BER performance of LMMSE and ACA-LMMSE estimation performed with the fixed filter previously depicted. In order to generalize the method, we keep the same WSSUS-based model with the same parameters as we used in the last subsection 5.3.

We observe that Figs. 9 and 10 are quite similar, i.e. the difference between the BER curves of LMMSE and ACA-LMMSE is less than 0.5 dB. This result shows that the performance of the method is the same with a time-varying and a static filter $\mathcal{G}$. 

26
5.5. *Practical application of ACA-LMMSE*

The proposed method used with a constant filter then allows to perform an efficient estimation with a low complexity. Moreover, such a filter can be simply designed with only delay lines. It then makes the method applicable with a low cost of implementation. Due to the assumption of a totally unknown channel in reception, this simple and efficient method could find some applications in many domains of telecommunications, such as the cognitive radio.

5.6. *Illustration of the Masking Effect*

This subsection aims at showing the efficiency of the proposed solution compared to the usual LMMSE in the case of physical channels having varying statistics parameters (see section 5.3), which is due to the masking effect of the artificial channel on the physical channel. The physical channel is built from *US Consortium*, but with parameters (number of paths, shape of the channel intensity profile and maximum delay) varying from a frame to another. The first frame is simulated using usual channel statistics of *US Consortium*. Fig. 11 shows the evolution of the BER in perfect estimation, ACA-LMMSE and LMMSE as a function of the time and for a $\frac{E_b}{N_0}$ ratio fixed at 25 dB. The parameters of LMMSE estimation are those of *US Consortium* during all the simulation duration. The parameters used to perform ACA-LMMSE estimation are: 20 artificial paths, a decreasing exponential profile and $\tau_{\text{max}}^{(G)} = T_{CP} = 5.33$ ms during all the simulation duration.

We observe that ACA-LMMSE is robust to the variations of the parameters of the physical channel and keeps its BER value around $10^{-3}$ over 17 frames. On the contrary LMMSE reaches only a BER value equal to $10^{-2}$ from the second frame (which tallies with the start of the statistics variations) to the last. This is due to the fact that usual LMMSE estimation method is badly adapted for channels having varying statistics, or its parameters must be updated. In
conditions of non-varying statistics of the channel (Figs. 5 and 9 and for the first frame), LMMSE and ACA-LMMSE BER performance almost matches. When the channel statistics change with time, the usual LMMSE is then no more adapted. On the contrary, ACA-LMMSE keeps its good performance of estimation with a low computing cost, using $\mathcal{G}$ which acts like a mask for $\mathcal{H}$ for any statistics variations.

5.7. Why not directly use LMMSE with filter parameters?

This subsection aims at proving the legitimacy of masking the physical channel by an artificial one to perform ACA-LMMSE estimation. Fig. 12 compares LMMSE and ACA-LMMSE BER, both performed with the parameters of the artificial channel: $L = 20$ artificial paths, a decreasing exponential profile and $\tau_{max}^{(G)} = 5.33$ ms. The goal is then to show the effect of LMMSE estimation performed with $R_{\mathcal{H}} = R_{\mathcal{G}}$, i.e. with parameters far from those of $\mathcal{H}$. The used channel is the US Consortium model and the robustness $C$ is chosen.

LMMSE estimation of the channel $\mathcal{H}$ performed with non-adapted parameters (the ones of $\mathcal{G}$) reaches an error floor equal to $7 \times 10^{-4}$. On the contrary, these parameters are adapted to ACA-LMMSE estimation, which requires to be performed with the ones of the artificial channel $\mathcal{G}$. We observe that the BER curve of ACA-LMMSE does not reach an error floor with the considered $E_b/N_0$ values. It proves the importance of the step of masking the unknown physical channel by an artificial one. Thus, Fig. 12 shows that LMMSE estimation requires an adapted covariance matrix to be performed. $R_{\mathcal{H}}$ being unknown, it justifies the use of an artificial channel whose covariance matrix is $R_{\mathcal{G}}$.

6. Conclusion

In this article we proposed an efficient method of channel estimation based on the usual LMMSE. The method is called artificial channel aided- (ACA-
LMMSE. The principle consists in introducing, at receiver side, an additional filter acting like an artificial channel and whose entries are pilots. From the estimator point of view, the received signal is the sum of the useful signal distorted by the physical channel and a pilot signal distorted by the artificial channel. As the filter is perfectly controlled by the user, its parameters are chosen to allow an LMMSE estimation of the sum of physical and artificial channel using only the covariance matrix of the artificial channel. The estimation of the physical channel is then performed by subtracting the known coefficients of the artificial channel. This effect of masking the channel by a filter is studied, and we have shown the conditions required on the parameters of the artificial channel to perform this mask effect. Furthermore, we have shown that the ACA-LMMSE method is efficient, even in condition of statistics variations of the physical channel. As a consequence, the matrix inversion for LMMSE needs to be computed only once during all the transmission duration, which reduces the complexity a lot, compared to usual LMMSE, which requires regular updates of the channel covariance matrix. Furthermore, we also reduce the complexity of ACA-LMMSE by using a constant filter without loss of channel estimation efficiency which makes the method applicable in practice. This method supposes a perfect synchronization between the useful signal and the artificial signal. Consequently, further work could concern this particular problem. The application of the method in cognitive radio can also be an interesting subject.

Appendix A

We develop (22) using the intensity profile (24) :
The developments are similar for the constant profile and the growing exponential profile.

In the case of constant profile, the term \( u - v \) appears in the denominator of \((\mathbf{R}_d)_{u,v}^{\text{dep}}\) in (28). However, \((\mathbf{R}_d)_{u,v}^{\text{dp}}\) is defined for \( u = v \). Indeed, the development of the first terms of the Taylor series of the exponential function in zero is \( e^x = 1 + x + o(x) \). Replacing the exponential by its development in zero for the case \( u = v \), it gives:

\[
(\mathbf{R}_d)_{u,u}^{\text{cp}} = DC.
\]  

The diagonal elements of the covariance matrix are then defined.

References


Table 1: Number of operations of ACA-LMMSE compared to usual LMMSE.

<table>
<thead>
<tr>
<th></th>
<th>Usual LMMSE</th>
<th>ACA-LMMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{II}$$(R_{II} + \frac{\sigma^2}{T}I)^{-1}$</td>
<td>$O(M^n)$</td>
<td>$O(M^n)$</td>
</tr>
<tr>
<td>$\tilde{R}_{II} = \tilde{H}_n^L S H_n^L$</td>
<td>unnecessary</td>
<td>unnecessary</td>
</tr>
</tbody>
</table>

Table 2: Parameters of US Consortium channel model.

<table>
<thead>
<tr>
<th>US Consortium</th>
</tr>
</thead>
<tbody>
<tr>
<td>path 1</td>
</tr>
<tr>
<td>delay</td>
</tr>
<tr>
<td>gain</td>
</tr>
<tr>
<td>Doppler spread</td>
</tr>
<tr>
<td>Doppler spectrum</td>
</tr>
</tbody>
</table>

Table 3: Parameters of Robustness C mode.

<table>
<thead>
<tr>
<th>Robustness C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol duration</td>
</tr>
<tr>
<td>CP duration</td>
</tr>
<tr>
<td>Frame duration</td>
</tr>
<tr>
<td>Number of carriers</td>
</tr>
<tr>
<td>Signal bandwith</td>
</tr>
<tr>
<td>Signal constellation</td>
</tr>
</tbody>
</table>

Figure 2: BER curve in function of $\tau_{max}^{(G)}/\tau_{max}^{(H)}$ and $E_b/N_0$.  

34
Figure 3: Normalized Error between the eigenvalues of $\mathbf{R}_K$ and $\mathbf{R}_G$.

Figure 4: Three profiles for $\Gamma(\beta)$.

(a) Decreasing exponential profile

(b) Constant profile

(c) Growing exponential profile
Figure 5: BER of the ACA-LMMSE estimator for three different intensity profiles.

Figure 6: Evolution of MMSE of ACA-LMMSE compared to LS and LMMSE as function of $\frac{P}{\sigma^2}$ for PB distribution.
Figure 7: Evolution of MMSE of ACA-LMMSE compared to LS and LMMSE as function of $P_\sigma^2$ for SP distribution.

Figure 8: BER of ACA-LMMSE compared to LMMSE and LS for US Consortium Channel.
Figure 9: BER of ACA-LMMSE compared to LMMSE in general channel model.

Figure 10: BER of ACA-LMMSE compared to LMMSE performed with a constant filter.
Figure 11: BER of ACA-LMMSE compared to LMMSE in condition of varying channel statistics.

Figure 12: BER of ACA-LMMSE compared to LMMSE performed with the parameters of the artificial channel.