Ultra-Fast Blind Equalization for OFDM: Principle and Steps Towards Implementation

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Abstract—Blind equalization allows the transmitted signals to maximize their spectral efficiency, but it is limited by the convergence time of used algorithms. In this paper, the authors propose a new frequency blind equalization method for orthogonal frequency division multiplexing (OFDM), which drastically reduces the transient state delay of the algorithm’s convergence process. It consists of reusing a proper number of received OFDM symbols that have been stored in a memory. Simulations results reveal that a quasi-instantaneous equalization process can be achieved. In addition, the implementation issues are discussed in this paper.

Keywords—Blind equalization, Constant Modulus Algorithm, OFDM.

I. INTRODUCTION

The cyclic prefix-orthogonal frequency division multiplexing (CP-OFDM) modulation scheme has been adopted in a large number of communications standards, mainly for its low-complexity implementation, and for its robustness against the multipath transmission channel. Furthermore, the use of pilot symbols allow the receiver to perform a simple channel estimation and inversion in the frequency domain [1]. However, these advantageous features are obtained at the cost of a loss of spectral efficiency. In order to enable higher data rate transmissions, pilots can be removed or the CP can be shortened. Such capabilities can be reached by means of blind equalization techniques.

It is proposed in [2] to use the feature of the CP to update the coefficients of the blind deconvolution filter in order to shorten the channel. This step is performed in [3] by minimizing the mean square error on the “artificially” null symbols between two consecutive subcarriers. Alternatively, the blind channel inversion can be carried out in the frequency domain by a one-tap per-carrier equalization as in [4] in which the simple decision directed algorithm is used. More recently, a maximum-likelihood (ML) based algorithm has been proposed in [5]. This blind equalizer achieves good performance, but it is particularly complex, especially if numerous subcarriers and high order constellations are considered.

The main drawback of blind equalization techniques is the convergence delay, as the received data during the transient state of the algorithm cannot be properly detected. In this paper, we propose a constant modulus algorithm (CMA) based method that drastically reduces the convergence delay of the blind equalizer. Unlike proposed techniques in the literature as in [6], [7] which mainly focused on the used cost function to improve the algorithms, the method presented in this paper consists of properly reusing the received OFDM symbols as input of the blind equalizer block. As a consequence, we have called the algorithm reuse of the input symbols-CMA (RIS-CMA). Obtained theoretical results show that this approach allows to asymptotically tend toward a quasi-instantaneous blind equalization process, which would significantly reduce the drawback of the blind channel inversion in comparison with pilot-aided methods. Furthermore, the implementation issue is investigated, therefore the expected delay gains are also investigated for a practical case.

The rest of the paper is organized as follows: Section II presents the structure of the blind OFDM receiver. The proposed algorithm is presented in Section III, and Section IV discusses the practical implementation of this method. Section V concludes this paper.

II. BLIND OFDM RECEIVER

A. System Model

In the considered system, the OFDM signal is assumed to be received with perfect time and frequency synchronization. Moreover, the cyclic prefix is supposed to be longer than the duration of the transmission channel, allowing to recover the orthogonality between subcarriers. Therefore, after the CP removal, the FFT of size \( M_{FFT} \), and the null subcarriers withdrawal, the \( n \)-th frequency component of the \( n \)-th OFDM symbol can be written as

\[ Y_{m,n} = H_{m,n}X_{m,n} + W_{m,n}, \quad (1) \]

where \( X_{m,n} \) and \( Y_{m,n} \) are the transmitted and received complex symbols respectively. It is assumed that a 16-QAM constellation is used. The subscript \( m \in \{0,1,..,M-1\} \) points out the subcarrier index, and \( M \leq M_{FFT} \) is the number of useful subcarriers after the \( M_{FFT} - M \) null subcarriers withdrawal. The variable \( W_{m,n} \) is the additive Gaussian noise sample with a variance \( \sigma^2 \), and \( H_{m,n} \) is the channel frequency response, which is supposed to be time-invariant. We also define \( T_s \) the duration of one OFDM symbol.

B. Frequency Blind Equalization

The equalization consists of canceling the channel in (1) in order to recover the symbols \( X_{m,n} \) from the received \( Y_{m,n} \).
To achieve this, a common solution consists of multiplexing pilots in the data stream in order to estimate and then invert the channel. These data-aided techniques are largely dealt with in the literature as in [1], [8] and [9], but they are out of the focus of this paper. The frequency blind equalization addressed here is hereafter proposed to reduce the convergence delay of the update algorithm (6) several times during a $T_e$ time.

It can be observed in Fig. 1 that the blind receiver is composed of two main blocks: i) the demodulator, whose processing time $T_{dem}$ is equal to $T_s$, and ii) the equalizer whose processing time $T_{eq}$ can be different from $T_{dem}$, since $M_{FFT} > M$ and since the two blocks in Fig. 1 can be processed with a different frequency. As a consequence, a latency between the processing of $Y_m$ and $Y_{m+1}$ could be used at the equalizer blocks to improve its convergence speed by reusing some previously received symbols $Y_{m'}$. In order to further describe the proposed method, some variables need to be defined:

- $K = \left\lfloor \frac{T_{dem}}{T_s} \right\rfloor \geq 1$ is the number of updates in (6) that the equalizer is able to perform during the duration $T_{dem}$, with $\lfloor \cdot \rfloor$ the floor function. This parameter is called reuse factor.
- $\Omega_{m,n}$ is the set of the signal samples received on the $m$-th subcarrier, i.e. $\Omega_{m,n} = \{Y_{m,0}, Y_{m,1}, \ldots, Y_{m,M-1}\}$.
- $\Psi_{\Omega_{m,n}}$ a given vector of size $1 \times (K-1)$ composed of elements belonging to $\Omega_{m,n}$, and $\psi_{\Omega_{m,n}}(k)$ the $k$-th element of $\Psi_{\Omega_{m,n}}$. The way to build $\Psi_{\Omega_{m,n}}$ is described afterward.

The principle of proposed method can be formalized for any subcarrier $m \in [0, \ldots, M-1]$ as follows:

**Step 1** At instant $n$, a vector $Y_{m,n,K}$ containing $K$ samples of the received signal is defined as

$$Y_{m,n,K} = [Y_{m,n}, \Psi_{\Omega_{m,n}}]$$

**Step 2** For any $n \in \mathbb{N}$ and $k \in [1, K-1]$, the update algorithm in (6) can be reformulated by using $Y_{m,n,K}$ instead of $Y_{m,n}$ using the following expression

$$F_{m,n}(k+1) = F_{m,n}(k) - \mu((Z_{m,n}(k))^2 - R)Z_{m,n}(k)Y_{m,n}^\ast,$$  

where $F_{m,n}(k)$ is the $k$-th updated coefficient at the $n$-th OFDM symbol, and $Z_{m,n}(k)$ is the corresponding output of the equalizer (the output symbols $Z_{m,n}(k)$ should be ignored after this step). Note that (8) is the same as (6) in the case $K = 1$.

**Step 3** Finally, after the $K$-th reuse, the output of the equalizer is such as

$$F_{m,n+1}(0) = F_{m,n}(K)$$

$$Z_{m,n} = F_{m,n}(K)Y_{m,n}.$$
Blind Equalization with RIS

We call the proposed method in (7)-(9) reuse of the input symbols using CMA blind equalization (RIS-CMA). This proposed method is described from step 1 to 3 in Fig. 2, by comparison with the usual CMA algorithm in (6). The way of choosing the elements \( \psi_{\Omega_{m,n}}(k) \) in the vector \( \Psi_{\Omega_{m,n}} \) is addressed hereafter.

It is worth noting in (6) that the coefficient \( F_{m,n} \) of the CMA method contains part of the previously received symbols \( Y_{m,n-1}, Y_{m,n-2}, \) etc., due to the iterative structure of the update algorithm. However, the transition from state \( n \) to state \( n+1 \) is probabilistic since the input symbols \( Y_{m,n} \) are uncorrelated. Therefore, the behavior of the RIS-CMA method in (7)-(9) could be close to CMA in (6) if the correlation between the elements \( \psi_{\Omega_{m,n}}(k) \) and \( F_{m,n} \) is minimized. The solution to achieve this behavior consists of choosing in the set \( \Omega \) the symbols \( \psi_{\Omega_{m,n}}(k) = Y_{m,n} \) that have been reused in the farthest past at each instant \( n \). Thus, the elements \( \psi_{\Omega_{m,n}}(k) \) can be chosen by means of the following procedure:

1) At iteration \( n \), we define the \( 1 \times Kn \) vector \( Y_{m,K} \) as the concatenation of the vectors \( Y_{m,n,K} \) as:
\[
Y_{m,K} = [Y_{m,n-1,K}, Y_{m,n-2,K}, \ldots, Y_{m,0,K}].
\] (10)

2) The \( \psi_{\Omega_{m,n}}(k) \) elements in \( Y_{m,n,K} \) are chosen as:
\[
\psi_{\Omega_{m,n}}(k) = Y_{m,K}(Kn+1-k).
\] (11)

Note that the main issues of the possible practical implementation of such a procedure (in term of memory) is discussed in Section IV. It must be emphasized that the principle of proposed method is independent from the adopted equalization algorithm, and therefore, could be easily used with CMA [7], MMA [6], or MMSE-based equalizers [3].

**B. Achieved Simulation Performance**

From the equalizer point of view, \( Kn \) updates of the coefficients \( F_{m,n} \) are performed by using RIS-CMA in (8)-(9), whereas only \( n \) updates are carried out with CMA in (6) at the \( n \)-th received symbol. Therefore, the convergence speed of RIS-CMA (in number of OFDM symbols) should be \( K \) times higher than CMA thanks to the reuse of the input symbols procedure. Furthermore, if the condition of the uncorrelation between the \( Y_{m,n} \) symbols is fulfilled, then the same steady-state performance for RIS-CMA as for CMA can be expected.

In order to validate the above mentioned comments, Fig. 3 shows the mean square error (MSE) defined by:
\[
MSE = E\{||Z_{m,n}||^2 - ||X_{m,n}||^2\}
\]
versus the number of OFDM symbols \( n \). The used transmission channel is the Proakis 1 described in [12], the constellation is 16-QAM, and \( M = 128 \). The trajectories of CMA and RIS-CMA are compared using different \( K \) values. Furthermore, the MSE obtained for the pilot-aided least square (LS) estimator has been plotted as reference. A preamble composed of \( M \) equipowered pilot tones such that \( ||X_{m,n}||^2 = 1 \) has been used. The LS estimated channel (see [9] for more details) is defined by:
\[
H_{m,n}^{LS} = \frac{Y_{m,n}}{X_{m,n}} = H_{m,n} + \frac{W_{m,n}}{X_{m,n}},
\] (12)
and a one-tap per carrier zero forcing (ZF) equalizer is utilized. The plotted curves reveal that the convergence duration of RIS-CMA is \( K \) times lower than CMA. Thus, it can be observed in Fig. 3 that CMA requires roughly 2000 OFDM symbols to reach its steady-state, whereas the proposed RIS-CMA with \( K = 32 \) achieves the same MSE performance by using only 62 iterations. It corresponds to a reduction of 99% of the transient state of the algorithm, which justifies the qualification of “ultra-fast” blind equalization. To the best of the authors’ knowledge, no blind equalization algorithm achieves such performance. Furthermore, it can be observed that the steady-state MSE performance is the same for both CMA and RIS-CMA equalizers.

Note that, from achieved results in Fig. 3 proposed RIS-CMA method is able to asymptotically reach a quasi-instantaneous convergence. It largely reduces the major drawback of blind equalization processes, in comparison with pilot-aided techniques. In fact, Fig. 3 shows that the RIS-CMA...
using $K = 32$ achieves a lower MSE performance than the pilot-aided ZF equalization with LS channel estimation after only several OFDM symbols. This reflects the fact that LS is not the optimal channel estimator in the mean square error sense. Besides, it must be reminded that in addition to achieved “ultra-fast” convergence speed, proposed method allows the transmitted signal to be more spectral efficient than signals including pilots. However, the results in Fig. 3 are theoretically obtained for any $K$. The following objective is to focus on proposed algorithm performance under realistic implementation constraints. This is deeply analyzed in the following section.

IV. DISCUSSION ON THE IMPLEMENTATION

It worth noticing that the symbol rate at the output of the OFDM demodulator is limited by the FFT processing speed (which must fulfill the timing constraints given by the symbol time $T_s$). The objective in this paper is to reuse $K$ times the received symbols during $T_s$ to increase the algorithm’s convergence speed.

Fig. 4 shows the proposed implementation of the blind equalization cell. OFDM symbols at the output of the FFT are collected at the dual-port memory with a depth $L$. Among these $L$ symbols, the earliest $K$ ones in the memory are used to update $F_{m,n}$. The processing performed by this cell consists mainly of a complex multiplication followed by an update of $F_{m,n}$, as defined in (8), in the register REG1. The output of this cell is read from the register REG4 at a rate of $1/T_s$. In REG2 and REG3 parameters $\mu$ and $R$ are stored respectively and are used to update $F_{m,n}$.

The memory depth $L$ is defined considering the compromise between the memory size and the minimization of the correlation between the OFDM symbols. Fig. 5 shows the trajectories (MSE versus OFDM symbols) of proposed RIS-CMA for $K = 8$, and using six different memory lengths ($L \in \{2, 8, 16, 24, 32, 40\}$). It should be noted that the definition of the $\psi_{m,n}(k)$ elements must be reconsidered by substituting $(k \mod L)$ instead of $k$ in (11), where $\mod$ denotes the modulo operation. Depicted results reveal that $L$ does not affect the convergence speed of the proposed method, but the optimal steady-state performance is nearly achieved by using $L \gtrsim 50$ (see Fig. 5). Other series of simulations have demonstrated that a rule of thumb can be derived as: for a set $K$ value, the best performance of RIS-CMA is achieved by using high values of $L$, independently of $K$. In particular, in order to achieve performance equivalent to that of pilot-aided LS estimator, the memory depth $L$ must be chosen as $L > 30$.

Table I depicts the impact of the parameter $L$ on the memory requirements for real setting design scenario. This table illustrates the required memory size (in kbits) for the RIS-CMA under 3GPP LTE [13] standard specification while considering 32 bit precision for the received complex valued symbols. In addition, Table I shows the minimum required number of block RAM (BRAM) when considering a Xilinx 7 Series FPGA [14]. Of course, the number of BRAM can be higher if higher read/write speeds are needed. The obtained results in Table I illustrate the limits of the proposed technique from memory requirements point of view. These limits are mainly the high memory requirements for large FFT size and high $L$ values. However, these issues are overcome using exist-
The reuse of the received symbols in accordance with tight timing constraints.

In this paper, we presented a new ultra-fast blind equalization technique RIS-CMA based on proper reuse of received symbols. Obtained results revealed that by using this technique, we can asymptotically reach a quasi-instantaneous convergence of the blind equalization algorithm. The implementation issues of proposed technique has been discussed in this paper. In particular, it has been shown that the memory depth $L$ must be chosen as $L > 30$ in order to achieve performance equivalent to that of pilot-aided LS estimator. Thus, the limits of the proposed technique from implementation point of view are mainly: the high requirements in terms of memory, and the operating clock frequency. However, the authors demonstrated that these issues are overcome using existing FPGAs and in the worst considered case, less than $10\%$ of the FPGA memory resources are used. However, the complexity of the blind equalization cell still can be reduced.

Our future work will focus on: i) Adapting the blind equalization algorithm to cover time-selective channels, ii) optimize the implementation of blind equalization cell to reduce the number of real multiplications.

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