Distributed predictive control for complex hybrid system. The Refrigeration System example

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Abstract: In this paper, a distributed model predictive control (DMPC) is applied to the supermarket refrigeration system benchmark specified by the HYCON project. This system is an example of continuous nonlinear system controlled by on/off variables. The aim of the proposed approach is to overcome the computational complexity of previous centralized MPC by distributing the control while keeping the communication among controllers at a low level. Two types of interactions among controllers are considered: for some variable the predicted values are broadcast to involved controller while for others a so-called price mechanism is used.

Keywords: Hybrid system, Predictive control, distributed control

1. INTRODUCTION

Supermarket refrigeration systems consist in a set of display cases located in the sales area connected to a rack of compressors and a condenser unit located in the technical area. The classic control of these units is performed locally: the temperature of the air inside each display is controlled by an hysteresis controller that switches an inlet valve admitting some refrigerant liquid inside the display and the pressure at the suction manifold is controlled by switching on and off some compressors. A limitation of this decentralized control is that the displays synchronize themselves then requiring important compressor power and frequent switching controls (Larsen et al., 2005). The use of more evolved control technics is appealing to avoid this synchronization and to improve the efficiency and the life-time of the system.

This continuous process is nonlinear with logical control inputs, leading to difficulties to design its controller and it has then been used to define a benchmark for hybrid control (Larsen et al., 2007). A first approach by Larsen et al. (2005) was based on the linearization of the equation and the use of the Mixed Logical Dynamical (MLD) framework (Bemporad and Morari, 1999) to solve the Model Predictive Control (MPC) problem. A second approach by Sonntag et al. (2008) is also based on MPC but the optimization is based on a nonlinear model. In order to decrease the complexity, low level controllers of the temperature are designed for each display and the parameters of these controllers are fixed by a high level NMPC that also controls the switching of the compressors. The optimization is performed by considering an increasing number of switches on the prediction horizon to reduce its logical part and its complexity. Finally, Sarabia et al. (2009) also propose a NMPC approach to control this system. In this proposition the compressors are traditionally controlled by a PI controller and the NMPC applies on the control of the display valves. In order to reduce the complexity of the optimization, the decision variables are changed from the logical on/off variables to real variables corresponding to the time when the state of a valve is changed.

These approaches achieve good results but they are based on a centralized structure that requires a global optimization for the MPC. A controller has to be computed for each configuration of displays and compressor rack and the complexity of the controller drastically increases with the number of display cases. The aim of this paper is to study if it is possible to achieve the same level of performances while using a decentralized control structure with a controller associated with each display case, in order to get a more flexible global control of the refrigeration system while keeping model predictive controllers as they are able to cope with the constraints and the various objectives of the system.

In distributed MPC, the system is decomposed into subsystems and a controller is associated to each sub-system. At each sample time, the local controller obtains measurements from its subsystem and information from other controllers, and then it solves an optimization problem on a finite horizon to determine its actions. As the optimal solution for each sub-system are linked to the actions of the others controllers, the DMPC structure depends on the information available from the other controllers at the first step and the communication during the second one (Negenborn et al., 2008). This leads to search for a compromise between the performances of the control, the complexity of the computation and the level of required communication. According to the structure of the system many solutions are considered. For example, for subsystems that are coupled by their controlled inputs and not by their state variables, Zhang and Li (2007) propose a control architecture where at each sample time each controller computes its optimal inputs then commu-
communicate it with other controllers and iterates the computation/communication phase until a consensus is reached. For the same type of systems, Maestre et al. (2009) propose that the controllers communicate also their criteria values in order that a suboptimal consensus is reached with only two communication phases. For serial systems where the output of a subsystem is the input of another one, it is possible to introduce two local views of the interaction variable and an extra variable aiming at constraining the equality of the local views. This extra variable may be considered as a Lagrange multiplier (Negenborn et al., 2008) or a price (Rantzer, 2009).

The next section briefly presents the supermarket refrigeration system and the control specification. For a more detailed specification of the plant and of the control the reader is referred to (Larsen et al., 2007) or (Sarabia et al., 2009). Section 3 describes the proposed control architecture based on the idea that the prediction phase of the MPC can be used to reduce the communication among the controllers. Section 4 provides some results and comparisons with classic control of the system. Finally conclusions are given in section 5.

2. SYSTEM DESCRIPTION - THE MAIN DIFFICULTIES

The simplified layout of the refrigeration system is represented on figure 1. A complete description is available in Sarabia et al. (2009). A bank of compressors compresses the low pressure refrigerant from the suction manifold. This high pressure refrigerant is condensed by the condenser unit and the liquid refrigerant can then be admitted into the parallel display cases under the control of switching inlet valves. The liquid evaporates inside the evaporator of the display case then cooling the air that circulates inside the display case and finally the goods. The evaporated refrigerant is then brought back to the suction manifold.

It is considered that the condenser unit and the liquid circuit have little effects on the global behavior and will not be considered. From the equations of the display cases, the suction manifold and the compressor rack that are detailed below it is possible to derive the data flow model of the system in figure 2. This schematic representation gives prominence to two classes of subsystem: on the one hand the display cases and the compression part on the other hand. These two subsystems interact each other by the mean of the refrigerant flow circulating from the display cases to the suction manifold and also by the mean of its pressure.

2.1 The display cases

As it can be seen on figure 2, each display case is considered as a dynamical system characterized by one discrete input \( e_i \), which is linked with the opening of the inlet valve, one interaction input \( p_{suc} \) which is the pressure in the suction manifold and one disturbance input \( q_{airload} \). The dynamics of this systems can be represented by a four dimensional state space corresponding to different temperatures in the display case. This leads to the following state equation:

\[
\begin{align*}
\dot{x}_1 &= A_1 X \\
\dot{x}_2 &= g_2(X, p_{suc}) \\
\dot{x}_3 &= A_3 X + q_{airload} \\
\dot{x}_4 &= \begin{cases} 
\frac{X_{AM} - x_4}{\alpha}, & \text{if } e_i = 1, \\
g_4(X, p_{suc}), & \text{if } e_i = 0 \text{ and } x_4 > 0, \\
0, & \text{if } e_i = 0 \text{ and } x_4 \leq 0,
\end{cases}
\end{align*}
\]

in which \( A_1 \) and \( A_3 \) are \( \mathbb{R}^{1 \times 4} \) matrices, \( g_2 \) et \( g_4 \) are nonlinear functions, \( \alpha \) and \( x_{AM} \) are positive constant. This is a nonlinear switched system, in which the switchings are orchestrated by the state vector and also the discrete input.

The output of the system, corresponding to the refrigerant flow, denoted with \( f_i \), is also a nonlinear function \( h \) of the state and the pressure \( p_{suc} \) given by:

\[
f_i = h(X, p_{suc}).
\]

2.2 The compression part

The second subsystem studied is the compression part. Its behavior depends on one discrete input \( Comp \), which is the number of compressors activated \( (\text{Comp} \in [0, n_c]) \), one interaction input \( f_{in} \), the sum of the refrigerant flow of the display cases, one disturbance input \( f_{ref} \) generated by
unmodelled entities. The flow pumped by the compressors is supposed to be proportional to the number of activated compressors:

\[ f_{\text{comp}} = \gamma \cdot \text{Comp}, \quad (3) \]

with \( \gamma \) a positive constant.

The state of the system is then composed of the pressure in the suction manifold and its evolution is given by the following differential equation:

\[ \frac{dp_{\text{suc}}}{dt} = f_{\text{in}} + f_{\text{refl}} - \frac{f_{\text{comp}} h_p(p_{\text{suc}})}{\beta h'_p(p_{\text{suc}})}, \quad (4) \]

where \( h_p \) and \( h'_p \) are nonlinear functions of \( p_{\text{suc}} \) linked to the refrigerant density and \( \beta \) is a positive constant.

The resulting subsystem is a dynamical hybrid system, more precisely it is a continuous nonlinear system governed by a discrete input.

2.3 Specifications et control objectives

The variables which have to be controlled are the temperature \( t_{\text{air}} = x_2 \) for each display case and the suction manifold pressure \( p_{\text{suc}} \). More precisely, the specifications are that each temperature must remain between 2 and 5 °C and the pressure must remain under \( p_r \), a time varying reference pressure.

The control objectives are then to keep the controlled variables within the specified bounds while minimizing the number of compressor switches, the number of switchings of the inlet valves and the power consumption. This last criterium is directly linked to the number of activated compressors.

3. CONTROL STRUCTURE

A predictive control structure has been chosen to control the refrigeration system. The previous objectives can be rewritten into the determination, at each sampling time, of the control sequence (opening of the inlet valves, number of activated compressors) which minimizes the following criterium:

\[ J = \sum_{i=1}^{n_d} J_{V_i} + J_C. \quad (5) \]

For each display case, the criterium \( J_{V_i} \) can be expressed as:

\[ J_{V_i} = \sum_{i=1}^{h} G(t_o[k + i]) + \alpha V \sum_{i=0}^{h-1} \left( e(k + i) - e(k + i - 1) \right)^2, \quad (6) \]

where the first term penalizes the overtaking of the temperature constraints and the second term is linked to the number of switchings. \( h \) is the prediction horizon and \( k \) the considered sampling time.

For the compression part, the criterium can be described by the following equation:

\[ J_C = \sum_{i=1}^{h} G(p_{\text{suc}}(k + i)) + \alpha_{CS} \sum_{i=0}^{h-1} \left( n_{\text{com}}(k + i) - n_{\text{com}}(k + i - 1) \right)^2 + \alpha_{CN} \sum_{i=0}^{h-1} n_{\text{com}}(k + i), \quad (7) \]

where the first term is linked to the pressure specification, the second one is linked to the number of switchings and the last one to the power consumption.

The optimization variables in these criteria are the boolean and discrete inputs sequences and their number grows exponentially depending on the number of display cases, the number of compressors \( n_d \) available and also the prediction length \( h \). Estimations of the size of this optimization problem will be given later for some particular configurations. To reduce this complexity, we propose a distributed control of this system: the resulting control structure is developed in the following.

The main objective of our approach is more the study of a distributed architecture than the control of each component, consequently we will assume that all the state variables and the disturbances are observable. Moreover, we will neglect the losses and delays linked to the communication protocol. Considering that the sampling period is large enough compared to the network performances, this assumption is not really a limitation. We also consider that all the different controllers share the same sampling time and the same prediction horizon.

Because of the additive nature of the criterium 5 and the particular structure of the refrigerant system (figure 2), it seems to be natural to consider a control structure in which each display case is governed by one specific controller and the rank of compressors by another one.

The refrigeration system appears as a structure in which the compression part provides different consumers (the display cases) with a refrigerant flow capacity. These interactions can be materialized by the pressure \( P_{\text{suc}} \) which is the same for all the consumers, and the refrigerant flow \( f_{\text{n}} \) which is the sum of the consumptions.

To favor the convergence of the distributed optimization to the global optimum (Rantzer (2009)), it is interesting to use a mechanism which considers explicitly these interactions. As the refrigerant flow traduces the interaction between the producer and all the consumers, a price agent can be introduced to take it in consideration.

The global structure of the distributed control is detailed in figure 3.

More precisely, the interactions between the subsystems are taken into account in the control by two different mechanisms:

- the predictions of the pressure calculated by the compressor controller is directly broadcasted to the display case controllers, by the mean of the vector \( P_{\text{suc}} \).
Fig. 3. Distributed structure

- a market price is introduce for the refrigerant flow : from the prediction of the flow consumption given by the display case controllers (vectors $F_i$) and a desired production by the compressor controller (vector $F_{in,com}$), a price (vector $P$) is established, price that acts on the behaviors of each controller. The agent price principle lies on the fact that the resolution of the optimization problem of the criterium (5) can be done as follows: first by introducing additional variables $F_{in,com}$ and $P$, and then by solving the problem composed of:
  - the minimization for each display case of the criterium:
    $\tilde{J}_{V_i} = J_{V_i} + P.F_i$, (8)
  - the minimization for the compressor controller of the criterium:
    $\tilde{J}_C = J_C - P.F_{in,com}$, (9)
  - the maximization, component by component of the criterium (10):
    \[ p(j) \cdot \left( \sum_i f_i(j) - f_{in,com}(j) \right) \] (10)

At each sampling time, there are several exchange steps between the different controllers. Their discussions have to converge to a consensus which should correspond to the optimal values of the interaction variables. On a theoretical point, the convergence of discussions to the global optimization point has been proved in the linear case without any constraints for a quadratic cost function, using a gradient algorithm Rantzer (2009).

3.1 Local control of the display case

The objective of this controller is to determinate, for each sampling time $k$, the optimal sequence of discrete input $\sigma = (e(k), e(k+1), \ldots e(k+h-1))$.

It is based on a algorithm which minimizes, at each discussion step, the criterium (8), according to the predicted pressure $P_{suc}$ and price $P$. This resolution consists in building the succession of predicted states for each possible control sequence, and then to choose the one that minimizes (8). As the control of the inlet valve is boolean, the complexity of this optimization is $2^n$. The optimization let estimate also the predicted consumption of the refrigerant flow $F_i$.

3.2 Local control of the compressors

The compressor controller has to determinate, for each sampling time $k$, the optimal sequence of discrete variables $\sigma = (n_{com}(k), n_{com}(k+1), \ldots n_{com}(k+h-1))$. To apply a distributed optimization, it is based on an algorithm which solves, at each discussion step, the minimization of the criterium (9), for a given predicted flow price, according to the control sequence and to the flow $F_{in,com} = (f_{in,com}(k), f_{in,com}(k+1), \ldots f_{in,com}(k+h-1))$ that the suction manifold can admit from the display cases.

This is a mixed optimization problem, whose complexity is about $(n_e + 1)^h$ admissible sequences, for which a $h$ dimensional optimization is required. In the following, the number of admissible sequences will be reduced to $3^h$, by authorizing no more than one switching between two step times.

3.3 Price agent

The last component of the control structure is the price agent which determinates, at each sample time, the price $P = (p(k), p(k+1), \ldots p(k+h-1))$, according to the flux $F_i$ predicted by the controllers of the display cases, and the desired $F_{in,com}$ of the compressor controller. This evolution from a negotiation step to the next one will be done on a gradient base, as follows:

\[ p(j) = p(j) + \alpha (\sum_i f_i(j) - f_{in,com}(j)) \] (11)

where $\alpha$ is an adjustable parameter.

3.4 Control algorithm

All the local controllers have been clarified. In this section we present formally the different steps of the control algorithm.

At each sampling time, the first step is the computation of the control of the compressor rack, based on the measures of the pressure and the refrigerant flow. Then the prediction of the suction pressure by the compressor controller is communicated to the display case controllers. Based on this pressure predictions and the price vector each display case controller compute its optimal output and the predicted refrigerant flow. For these two steps, at the first iteration the price vector computed at the previous sampling time is used. Then the flow requirement from the compressor rack controller and the estimated flows by the display case controllers are communicated to the flow market that computes a new vector price using a gradient approach. If there is an agreement on the flows, the iteration is stopped and the control outputs are sent to the plant. As there is no convergence guarantee, a maximum number of iterations is imposed, which also stops the negotiation if it is reached.
Supermarket Refrigeration System

The proposed distributed controller has been tested using 4.1 Presentation of the different tests

D during the day (resp. during the night) and N respect during the day (resp during the night), D to 1 q are stopped so mode the display cases are closed and other equipments 0 q cases are open and some others equipments consume some The system has two modes. In Day is composed of six compressors and ten display cases. In the second one, it first one, the refrigeration system is constituted of three compressors and three display cases. The two scenarios used to and Matlab/Simulink on a configuration with three display cases and three compressors. The parameters have to be defined for the MPC algorithm. First, the general behavior can be modified by changing the weights of the terms in the criterions, and then a compromise has to be done between the number of switching, the constraints respect and the energy consumption. For instance, increasing the value of αV in equation (6) will reduce the number of switchings of the inlet valve, leading to wider range of temperature and an increased required power. Another family of parameters are linked to the MPC structure and concerns the sampling time and the prediction horizon. These parameters are directly related to the computation complexity. Indeed, the choice of the length of the prediction horizon is crucial, as the prediction has to be large enough to anticipate the switchings, but the minimization complexity is directly linked to this length.

The results obtained with the algorithm 3.4 are presented qualitatively in the figure 5. The respect of the constraints can be noticed, without the necessity to use the three compressors. To illustrate the relation between the price, the pressure and the valves, a more detailed view is given in the figure 6. As we can see, the price is directly linked to the pressure, and it growths as the pressure is near its

4.2 First configuration : 3 display cases et 3 compressors

As it is specified in the previous sections, different parameters have to be defined for the MPC algorithm. First, the general behavior can be modified by changing the weights of the terms in the criterions, and then a compromise has to be done between the number of switching, the constraints respect and the energy consumption. For instance, increasing the value of αV in equation (6) will reduce the number of switchings of the inlet valve, leading to wider range of temperature and an increased required power. Another family of parameters are linked to the MPC structure and concerns the sampling time and the prediction horizon. These parameters are directly related to the computation complexity. Indeed, the choice of the length of the prediction horizon is crucial, as the prediction has to be large enough to anticipate the switchings, but the minimization complexity is directly linked to this length.

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1 The SRS simulation kit has been developed by the University of Valladolid and offers an EcoSimpro simulated model of the system that can be embedded in Matlab/Simulink. It is available on http://astwww.bci.tu-dortmund.de/hycon4b/

2 All these details are explained in Sarabia et al. (2009)
Table 1. Quantitative comparison of the results

<table>
<thead>
<tr>
<th>Control</th>
<th>$D_{cst}$</th>
<th>$N_{cst}$</th>
<th>$D_{sw}$</th>
<th>$N_{sw}$</th>
<th>$D_{Pw}$</th>
<th>$N_{Pw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic</td>
<td>1.77</td>
<td>2.01</td>
<td>0.11</td>
<td>0.03</td>
<td>1.40E4</td>
<td>1.30E3</td>
</tr>
<tr>
<td>Decentralized</td>
<td>1.7E-6</td>
<td>1E-7</td>
<td>0.18</td>
<td>0.11</td>
<td>1.36E4</td>
<td>1.30E3</td>
</tr>
<tr>
<td>DMPC1</td>
<td>4.6E-6</td>
<td>3.9E-7</td>
<td>0.065</td>
<td>0.037</td>
<td>1.38E4</td>
<td>1.39E3</td>
</tr>
<tr>
<td>DMPC2</td>
<td>2.4E-6</td>
<td>5.5E-6</td>
<td>0.058</td>
<td>0.044</td>
<td>1.39E4</td>
<td>1.21E3</td>
</tr>
<tr>
<td>DMPC8</td>
<td>7.8E-6</td>
<td>2.4E-6</td>
<td>0.069</td>
<td>0.045</td>
<td>1.36E4</td>
<td>1.29E3</td>
</tr>
</tbody>
</table>

Fig. 5. Results for 3 display cases and 3 compressors, $N_{iter} = 2$.

bound. In this situation, we can notice fast switchings for the inlet valves of the display cases. Quantitatively, the table 1 presents the values of the performance indexes. Several tests have been made, varying the maximum number of iterations allowed (1 iteration (DMPC1), 2 iterations (DMPC2) et 8 iterations (DMPC8)). No significant difference can be noticed between the three experiences. A more precise analysis of the number of iterations used to converge exhibits a mean number of 2.2, and in most cases, one iteration is enough.

For 3 display cases, 3 compressors and a prediction horizon of 8 steps, the computation complexity required at each sample time in a centralized approach would be near $(3 \cdot 2^8) = 1.1E11$ possibilities, which could be difficult to obtain in real-time. In our approach, there are $3 \cdot 2^8 = 768$ cases to test for the display cases, and $3^3 = 6561$ optimizations for the compressors, which is considerably much less, even if the number of negotiation step is important. Otherwise, a comparison with a decentralized structure (no communication between the controllers) has also been done. This leads to the same results for the constraint respect and the power consumption, but the results have been obtained with much more switchings.

If a centralized controller could be considered for the previous configuration, in this case the combinatory explosion would lead to compute about $(6 \cdot 2^{10})^8 = 2E30$ possibilities ! Whereas in our method, the complexity is linear with respect to the number of display cases, which leads to $10 \cdot 2^8 = 2560$ possibilities for the display cases, and the same number $3 \cdot 2^8 = 768$ for the compressors, because of the assumption of only one switching allowed between two prediction times. The results are presented in the figure 7, in which the constraint respect can be noticed. The quantitative results are given in the table 2.

Fig. 6. Interactions between temperature, pressure and price.

Table 2. Quantitative results, for 10 display cases

<table>
<thead>
<tr>
<th>Control</th>
<th>$D_{cst}$</th>
<th>$N_{cst}$</th>
<th>$D_{sw}$</th>
<th>$N_{sw}$</th>
<th>$D_{Pw}$</th>
<th>$N_{Pw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic</td>
<td>1.6</td>
<td>1.9</td>
<td>0.25</td>
<td>0.78</td>
<td>1.9E4</td>
<td>4.6E3</td>
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<tr>
<td>Decentralized</td>
<td>1.9E-3</td>
<td>1.6E-3</td>
<td>0.44</td>
<td>0.30</td>
<td>1.9E4</td>
<td>4.3E3</td>
</tr>
<tr>
<td>DMPC2</td>
<td>5.5E-4</td>
<td>2.8E-4</td>
<td>6.7E-2</td>
<td>6.4E-2</td>
<td>1.9E4</td>
<td>4.6E3</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this paper, a distributed model predictive control of the refrigeration system is presented. The main advantage of this approach is that each local controller is simple and the complexity of the global controller is then linear with respect to the number of display cases. The number of required iterations at each sampling time may be limited without any severe degradation of the performances.

From a more theoretical point of view, the convergence of the distributed controller and the analysis of the closed-loop behavior have to be studied, as there is no formal guarantee at the moment.
Fig. 7. Qualitative results for 10 display cases and 6 compressors

REFERENCES


