Descriptor Systems for the Knowledge Modelling and Simulation of Hybrid Physical Systems

Les Systèmes Descripteurs pour la Modélisation et la Simulation de Systèmes Physiques Hybrides

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ABSTRACT. Hybrid systems considered in this paper are systems involving continuous and discrete phenomena. These systems can be represented by a discrete event model interacting with a continuous model, and the interface by ideal switching components which modify the topology of the continuous part at switching time. This paper deals with the knowledge modelling of the continuous part of such systems using the bond graph approach. The objective is to show the interest of the implicit representation : - to derive a unique implicit state equation with jumping parameters, - to derive the implicit state equation with index of nilpotency one corresponding to each configuration, - to analyse the properties of these models and - to compute the transitions.

RÉSUMÉ. Les systèmes hybrides étudiés dans cet article sont des dispositifs présentant des phénomènes événementiels associés à des phénomènes continus. Un tel système peut être représenté par un modèle événementiel en interaction avec un modèle continu, et l'interface par des interrupteurs idéaux qui modifient la topologie de la partie continue au moment de la commutation. Cet article traite de la modélisation de connaissance de la partie continue de ces systèmes grâce au formalisme des bond graphs. On y met en évidence l'intérêt de la représentation implicite : - pour déterminer un modèle d'état unique avec saut sur un nombre réduit de paramètres, - pour déterminer une équation d'état d'indice de nilpotence un pour chaque configuration, - pour analyser les propriétés de ces modèles, - et enfin pour calculer les transitions.

MOTS-CLEFS : Systèmes Hybrides, Systèmes Descripteurs, Systèmes Implicites, Modélisation, Bond Graphs.

KEY WORDS : Hybrid systems, Descriptor systems, Implicit systems, Modelling, Bond Graphs.
1. Introduction

Different models are proposed for the study of hybrid systems which are composed of interacting continuous and discrete parts as for instance in [BRA 95]. Generally, the discrete part is modelled by an automaton, and the continuous part is modelled by different sets of state equations according to the discrete state. Jump functions can be added to represent the discontinuities. However, no indication is given about the relations between the different models and the different jump functions.

This paper deals with the knowledge modelling for physical systems with switching devices. Switching devices which change the structure of systems are very common in the field of electricity (e.g., diodes, relays ..) and also in other fields (e.g., clutches, valves ...). They are the interface between the discrete (control) part and the physical continuous part.

Various researchers have studied this problem for which there are two main approaches: ideal switches with variable circuit topology, [LOR 93], [BUI 93], [SOD 95], and non-ideal switches with constant circuit topology, [DAU 93]. This second approach is a way to solve the problems for the simulation but is not applicable to the study of the model of systems with ideal switches [STR 94]. The first approach which is used here, permits both a simplified analysis with ideal elements when considering the global performance of the system and a more precise analysis if a more realistic model of the switching device is known.

The paper is organised as follows: as the bond graph formalism is used here to derive implicit junction structure matrixes, a brief introduction to this formalism is given in section 2. In section 3, the notion of implicit junction structure matrixes for switching bond graphs is introduced. A general state equation is derived from this representation in section 4 and, in section 5, for the linear case, the properties of the model are studied and the discontinuities are computed. In section 6, transition matrixes are proposed to compute the new implicit junction structure matrixes after a commutation. A simple example illustrates the different points in section 7. Before concluding, in the last section, the approach is applied to the simulation of an asynchronous motor with its starter.

2. Basic notions of bond graphs

The bond graph formalism [KAR 90] allows with a unique approach to model engineering components of different fields (mechanics, electricity, hydraulics ...). A first step consists in a breaking down of a system on the basis of power transfer between subsystems. This power transfer is graphically described by bonds.
A half arrow is used to indicate the sign convention for power. On figure 1, the power is positive when flowing from A to B. Power is the product of two variables: effort (e) (voltage in the field of electricity and force in the field of translation mechanics) and flow (f) (current in the field of electricity and velocity in the field of translation mechanics). The integral of those power variables, which are called energy variables, are respectively the moment (p) and the displacement (q).

2.1. *The basic elements*

The construction of the system is obtained by successive breakdowns until elementary components are obtained. These components can be grouped in four classes according to their behaviour versus energy: *power sources, storage elements, power dissipative elements, instant power transfer elements.*

There are two kinds of power sources: the *effort sources* Se and the *flow sources* Sf.

The storage elements are
- the *inertial elements* I described by a relation \( p = \Phi_I(f) \) (inductance in the field of electricity and mass in the field of translation mechanics),
- the *capacitive elements* C described by a relation \( q = \Phi_C(e) \) (capacity in the field of electricity and spring in the field of mechanics).

The power dissipative elements are called *resistive elements* R. They are described by a relation \( e = \Phi_R(f) \) (resistance in the field of electricity and friction in the field of mechanics).

The instant power transfer elements are used to associate the previous elements in order to build complex systems: 0 and 1 *junctions*, the *transformers* TF and *gyrators* GY:
- The 0 junctions correspond to parallel association in the field of electricity: the effort is the same on all the bonds.
- The 1 junctions correspond to serial association in the field of electricity: the flow is the same on all the bonds.
- The transformers correspond to ideal transformer in the field of electricity. It is characterised by a double relation: \( f_2 = n.f_1 \) and \( e_1 = n.e_2 \).
- The gyrators are characterised by a double relation: \( e_2 = r.f_1 \) and \( e_1 = r.f_2 \). They are used to describe electromechanical phenomena.

With the bond graph technique ideal switches can be modelled by zero effort (flow) sources when they are in one state and by zero flow (effort) sources when they are in a second state. If a switch is not ideal, it can be modelled by an ideal switch associated with other elements (R, I, C) on a 1 or 0 junction.
A general bond graph can always be represented by the diagram of figure 2 where a field with the switching components is distinguished. The convention for the direction of the power is shown on figure 2, i.e. from the sources towards the junction structure or from the junction structure towards the I, C and R elements.

2.2. The causality

In order to derive the mathematical model, a fundamental concept, called causality is used. The causality indicates for instance if a resistive element will be described by the relation \( e = \Phi(R)f \) or \( f = \Phi(R)^{-1}(e) \). It is represented by a causality stroke which indicates the element imposing the flow and the one imposing the effort (figure 3). On figure 3-a A imposes the effort on B and B imposes the flow on A, while on figure 3-b, on the contrary, B imposes the effort on A and A imposes the flow on B.

There are a priori two possibilities for a stroke to be placed on a bond, but the two elements to which this bond is connected can bring some restrictions. For sources, there is only one possibility : a flow source imposes a flow and an effort source imposes an effort. If the element is an energy storage element, there is one affection called derivative causality and an other one called integral causality.
Figure 4 illustrates this notion in the case of a capacitive element. Figure 4-a presents a capacity in derivative causality and the corresponding block diagram, whereas figure 4-b presents the case of integral causality. The type of causality is related to the mathematical expression in the block diagram. The use of the integral causality as often as is possible leads naturally to the state equations.

A coherent assignment procedure, searching for a maximum number of elements in integral causality, allows to affect the causality to each bond of the graph.

3. An implicit formulation of the junction structure relation

The causality assignment can be done for any initial acceptable configuration of the switches. The bond graph is then equivalent for one configuration to the block diagram represented on figure 5, where the following key variables are used:

- the state vector $X_i$ is composed of the energy variables in integral causality (p on I elements and q on C elements), and the complementary state vector $Z_i$ is composed of power variables (e on C elements and f on I elements);
- the semi-state vector $X_d$ is composed of the energy variables in derivative causality, (p on I elements and q on C elements), and the complementary state vector $Z_d$ is composed of power variables (e on C elements and f on I elements);
- the vectors $D_i$ and $D_o$ represent the variables going out of and into the R field;
- the vector $U$ is composed of the variable imposed by the sources, and $V$ with the complementary variables (flows in the effort sources and effort in the flow sources)
- $T_i$ is composed of the variables imposed by the switches in this initial acceptable configuration: the flow for the switches which are open and the effort for the switches which are closed;
- $T_o$ is composed of the variables in the switches: the effort for the switches which are open and the flow for the switches which are closed.
If we suppose that there is no causal loop with a unity gain [DIJ 91], a linear relation between the output of the junction structure $z_o = \left( \dot{X}_i, \dot{Z}_d, D_o, -T_o - V \right)^T$ and the input $z_i = \left( Z, \dot{X}_d, D, T, U \right)^T$ can be found:

$$z_o = J z_i,$$

where the matrix

$$J = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ -S_{31} & -S_{32} & -S_{33} & -S_{34} & -S_{35} \\ -S_{41} & -S_{42} & -S_{43} & -S_{44} & -S_{45} \\ -S_{51} & -S_{52} & -S_{53} & -S_{54} & -S_{55} \end{pmatrix}$$

is skew symmetric because $z_o^T z_o$ which represents the power flowing out of the junction structure is null.

Omitting the last row, this relation leads to the following implicit equation that we call the standard implicit form:

$$M \dot{X} = SW,$$  \hspace{1cm} (1)

with

$$\dot{X} = \begin{pmatrix} \dot{X}_i \\ \dot{X}_d \end{pmatrix}, \quad W = \begin{pmatrix} Z_i & Z_d & D_i & D_o & T_i & T_o & U \end{pmatrix},$$

$$M = \begin{pmatrix} 1 & -S_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -S_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & -S_{12} & 0 & 0 & 0 & 0 \end{pmatrix} S = \begin{pmatrix} S_{11} & 0 & S_{13} & 0 & S_{14} & 0 & S_{15} \\ -S_{12} & -1 & 0 & 0 & S_{24} & 0 & S_{25} \\ -S_{12} & 0 & S_{33} & -1 & S_{34} & 0 & S_{35} \\ S_{14} & 0 & S_{44} & 0 & S_{44} & -1 & S_{45} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$  \hspace{1cm} (2)
I and 0 denote structural identity and zero matrices, and the other elements are composed of 0, ±1, and the coefficients of the gyrators and transformers. The first row of this relation defines $\dot{X_i}$, the second one $Z_d$, the third one $D_o$ and the last one $T_o$.

$S_{11}$, $S_{33}$ and $S_{44}$ are skew symmetric. $S_{23} = 0$ because there is no relation between $Z_d$ and $D_i$ (or $D_o$ and $\dot{X}_d$). If it were the case, we could reverse a causal path between an element in derivative causality. As a consequence, the element in derivative causality would recover the integral causality. For the same reason $S_{22} = 0$, there is no relation between $\dot{X}_d$ and $Z_d$.

When the switches commutate, the causality of the corresponding elements changes and it is necessary to extend this change of causality to other bonds. Some energy storage elements can lose or recover the integral causality and the state equation is changed.

At this point it is very important to note that as the formal relations are not changed, the implicit equation that was found for one configuration of the switches remains true whatever the state of the switches is as long as no value is affected to $T_i$ or $T_o$. As we will see in section 4, this expression permits to compute a unique implicit state equation. If the system is not in the initial configuration, the elements of $T_i$ corresponding to the switches which are not in their initial state will no longer be null but the corresponding elements of $T_0$ will then be null.

For the different acceptable configurations of the switches, such a standard implicit form can be found. Some elements of the vectors $W$ and $X$ can permute, and the elements of the matrices $M$ and $S$ change, but the structural zero (0) or identity (I) matrices will always be the same. As it will be seen in section 5, this form is really convenient to compute the state equation in the initial configuration because in the ideal case $T_i = 0$.

4. Unique state equation formulation

In this section, considering the linear case, a unique state equation with jumping parameters is established. In the linear case, the fields are described by linear laws:

$$D_i = LD_o$$

for the resistive R field and

$$\begin{pmatrix} Z_i \\ Z_d \end{pmatrix} = \begin{pmatrix} F_i & F \\ F^T & F_d \end{pmatrix} \begin{pmatrix} X_i \\ X_d \end{pmatrix}$$

for the inertial I and capacitive C fields, where $\begin{pmatrix} F_i & F \\ F^T & F_d \end{pmatrix}$ is definite positive.
The third line in equation (1) is used to solve the algebraic loop due to causality coupling between the resistive elements \( (S_{33} \neq 0) \) analytically. Admitting that \((I - S_{33}L)\) is invertible leads to

\[
D_k = (I - S_{33}L)^{-1}\left(-S_{33}^T Z_i + S_{33} T_i + S_{33} U\right).
\]

The constitutive law of the storage elements can be used to eliminate \(Z_i\) and \(Z_d\).

Denoting \((I - S_{33}L)^{-1}\) and \(K = (S_{33} - S_{33}HS_{13}^T)\), we get the implicit equation:

\[
\begin{pmatrix}
1 & -S_{12} \\
0 & 0 \\
0 & -S_{24}^T
\end{pmatrix}
\begin{pmatrix}
\dot{X}_i \\
\dot{X}_d
\end{pmatrix} =
\begin{pmatrix}
S_{11} - S_{13}HS_{13}^T & 0 & S_{14} + S_{13}HS_{34} & 0 & Z_i \\
-S_{12} & -I & S_{24} & 0 & Z_d \\
S_{14} - S_{34}HS_{13}^T & 0 & S_{44} + S_{34}HS_{34} & -I & T_i \\
0 & 0 & 0 & T_o
\end{pmatrix}
\begin{pmatrix}
S_{15} + S_{13}HS_{35} \\
S_{25} \\
S_{45} + S_{34}HS_{35} \\
0
\end{pmatrix} + \begin{pmatrix}
U \end{pmatrix}
\]

which is not a state equation due to the presence of \(T_i\) and \(T_o\) whose entries are inputs or outputs according to the configuration of the switches. To derive a state equation a fourth line has to be added to take into account the constraint on the switches. We obtain the standard implicit state equation:

\[
E\dot{X} = AX + BU \tag{4}
\]

where

\[
X = \begin{pmatrix} X_i & X_d & T_i & T_o \end{pmatrix}^T,
\]

\[
E = \begin{pmatrix} 1 & -S_{12} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -S_{24} & 0 \\
0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} S_{15} + S_{13}HS_{35} \\
S_{25} \\
S_{45} + S_{34}HS_{35} \\
0 \end{pmatrix},
\]

\[
A = \begin{pmatrix} KF_i & KF \\
-S_{12}F_i - F_i^T & -S_{12}F - F_d \\
(S_{14} - S_{34}HS_{13}^T)F & (S_{14} - S_{34}HS_{13}^T)F - S_{44} + S_{34}HS_{34} - I \\
0 & 0 \end{pmatrix},
\]

In the initial configuration \(TT = 0\) and the fourth line leads to \(T_i = 0\). In an other mode, \(TT\) is a diagonal matrix whose elements are all null except those corresponding to the switches which are in a different mode than in the initial configuration. The fourth line leads to \((I - TT)T_i = 0\) and \(TT_0 = 0\).
This model (4) has been studied by several authors such as [DAI 89] and [CAM 92]. It is an association of dynamical and algebraic modes and it can exhibit impulsive and discontinuous phenomena at the origin. One of the main results is the following:

If \( \dim(X) = n \), and \( \text{rank}(E) = q < n \), then there are \( q \) dynamical modes and \( n - q \) algebraic modes. Moreover, if \( \text{rank}(E) = n + q - h \) then among the dynamical modes, there are \( q - h \) finite modes and \( h \) infinite modes.

The advantage of this form is that it is very synthetic. The commuting system is modelled by a unique state equation with a jump on a small part of its parameters (TT). However, the index of nilpotency is greater than one and this form is not convenient for simulation.

5. Properties of the state model

In this section, it is assumed that the system is in the initial acceptable configuration corresponding to \( TT = 0 \). The properties of the implicit state equation are studied as well as the transition with the previous configuration [BUI 97].

5.1. Equivalent explicit state equation

If the system is in the initial acceptable configuration, \( T_i = 0 \) and the state equation (4) can be simplified as follows:

\[
\begin{aligned}
& \begin{bmatrix}
I - S_{12} & 0 \\
0 & 0 \\
0 - S_{24}^T & T_{s_i}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_i \\
\dot{X}_d
\end{bmatrix} =
\begin{bmatrix}
K_F & K_F \\
-S_{12}^T F - F^T - S_{12}^T F - F_d \\
H_F & H_F - I
\end{bmatrix}
\begin{bmatrix}
X_i \\
X_d \\
T_s
\end{bmatrix} +
\begin{bmatrix}
S_{15} + S_{14}HS_{15} \\
S_{25} \\
S_{45} + S_{14}HS_{15}
\end{bmatrix}
U.
\end{aligned}
\]

or if we are not concerned with the switches variables

\[
\begin{aligned}
& \begin{bmatrix}
I - S_{12} & 0 \\
0 & 0 \\
0 - S_{24}^T & T_{s_i}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_i \\
\dot{X}_d
\end{bmatrix} =
\begin{bmatrix}
K_F & K_F \\
-S_{12}^T F - F^T - S_{12}^T F - F_d \\
H_F & H_F - I
\end{bmatrix}
\begin{bmatrix}
X_i \\
X_d \\
T_s
\end{bmatrix} +
\begin{bmatrix}
S_{15} + S_{14}HS_{15} \\
S_{25} \\
S_{45} + S_{14}HS_{15}
\end{bmatrix}
U.
\end{aligned}
\]
This equality means that in the case of hybrid physical systems, \( h = 0 \) which implies that state variables are not impulsive on commutation. This conclusion is consistent with the physical principles: the power variables \( \varphi \) and \( q \) cannot be impulsive. If the same rank test is done on the matrices from equation (5), the same number of dynamical mode with finite value is found, but algebraic and impulsive modes corresponding to \( T_o \) are added. The number of impulsive modes is \( h = \text{rank}(S_{24}) \).

It is usual when analysing the properties of (5) or (6) to multiply it by a non-singular matrix \( P \), and to operate a variable change \( Q \) to obtain a new equivalent equation:

\[
(P EQ)(Q^{-1}\dot{X}) = PAQ(Q^{-1}X) + (PB)U.
\]

To go further in the analysis of the implicit equation, let's multiply (6) by the non-singular matrix

\[
P = \begin{bmatrix}
I - K(F + F_{12}) & I \\
0 & I
\end{bmatrix}
\]

and do the variable change

\[
Q = \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = \begin{bmatrix}
X_i \\
X_d
\end{bmatrix},
\]

where

\[
Q = \begin{bmatrix}
I & S_{12} \\
0 & I
\end{bmatrix} \begin{bmatrix}
I \\
-R(S_{12}F + F^T)
\end{bmatrix}.
\]

We get the following implicit state equation:

\[
\begin{bmatrix}
I \\
0 
\end{bmatrix} \dot{X}_1 = \begin{bmatrix}
I \\
0 
\end{bmatrix} \dot{X}_2 + \begin{bmatrix}
K F_{12} \left( S_{12}^T F + F^T \right) & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
X_i \\
X_d
\end{bmatrix} + \begin{bmatrix}
S_{15} + S_{15}^T H S_{35} + K \left( F + F_{12} \right) R S_{25} \\
-R S_{25}
\end{bmatrix} U.
\]

This implicit state equation with an index of nilpotency equal to one is equivalent to two independent equations:
Descriptor systems for the knowledge modelling

- an ordinary state equation:
\[
\dot{X}_1 = \left(KF_{1} - K(F + F_{1}S_{12})B\right)(S_{12}F_{1} + F^{T})X_1 + (S_{13} + S_{1}HS_{35} + K(F + F_{1}S_{12})RS_{25})U
\]
where the state \( X_1 \) is continuous at the origin,
- an algebraic equation:
\[
X_2 = RS_{25}U
\]

5.2. Discontinuities on the variables at the commutation

At the initial time which is the commutation time, \( X_1^+ = X_1^- \) and \( X_2^+ = RS_{25}U \).

Using the previous variable change \( Q \), it is easy to compute the discontinuity on the original state variables:
\[
X_1^+ = X_1^- + S_{12}(X_d^+ - X_d^-)
\]
and
\[
X_d^+ = R\left(S_{12}F_{1} + F^{T}\right)(S_{13}X_1^- - X_d^-) + S_{25}U
\]

5.3. Amplitude of the pulses at the commutation

Another approach to describe the commutation is to use the implicit equation (1) (or (3)) of the system after the commutation. In this equation, \( T_0 \) can be impulsive on commutation:
- a switch which doesn’t commutate can have an impulsive flow if it was closed (and its effort stays null) or an impulsive effort if it was opened (and its flow stays null);
- a switch which commutates can have an impulsive flow if it closes (and its effort becomes null) or an impulsive effort if it becomes open (and its flow becomes null).

so \( T_0 = \delta(t - t_c) \). The integration of the first and third lines of (1) gives:
\[
\begin{pmatrix}
1 & -S_{12} \\
0 & -S_{25}
\end{pmatrix}
\begin{pmatrix}
X_1^+ - X_1^- \\
X_d^+ - X_d^-
\end{pmatrix} = \begin{pmatrix} 0 \\ -T \end{pmatrix},
\]
and the second line of (1) is the constraint:
\[
-S_{12}Z_1^+ - Z_d^+ + S_{25}U = 0.
\]
Combining (9) and (10) we obtain the same results for the discontinuity (7) and (8), and the amplitude of the pulses in the switches is:
\[
T = S_{25}^T\left(X_d^+ - X_d^-\right).
\]

In the non-linear case, (9) and (10) are still valid but the constitutive laws of the storage field can be non linear. In the general case, there is no explicit solution and a
numerical algorithm has to be used to solve (9) and (10) with the constraint
\[
\begin{pmatrix}
Z_f \\
Z_d
\end{pmatrix} = F
\begin{pmatrix}
X_f \\
X_d
\end{pmatrix}.
\]

6. The transition matrix

The preceding sections have shown the interest of the implicit standard form to derive state equations and the transitions. This section indicates the procedure to apply in order to recover this standard implicit form after a commutation. The case with the commutation of only one switch is first considered. Let's isolate the variables corresponding to the switch which commutates. The standard implicit form becomes:

\[
\begin{pmatrix}
I & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & I
\end{pmatrix}
\begin{pmatrix}
X_f \\
X_d
\end{pmatrix}
= 
\begin{pmatrix}
S_{11} & 0 & S_{14} & 0 & c_{14} & 0 & 0 & S_{15} \\
-s_{12} & 1 & 0 & 0 & S_{24} & s_{24} & 0 & 0 & S_{25} \\
-s_{13} & 0 & S_{34} & -1 & S_{34} & s_{34} & 0 & 0 & S_{35} \\
S_{14} & s_{14} & 0 & S_{44} & s_{44} & -1 & 0 & S_{45} \\
c_{14} & 0 & s_{14} & 0 & -s_{44} & 0 & 0 & -1 & s_{45}
\end{pmatrix}
\begin{pmatrix}
Z_f \\
Z_d \\
D_f \\
D_d \\
T_f \\
T_d \\
t_1 \\
t_2 \\
t_1^{T} \\
t_2^{T}
\end{pmatrix}
\]

where \(t_1\) and \(t_2\) are the variables associated to the switch which commutates. The commutation corresponds to the permutation of these two elements:

\[
\begin{pmatrix}
I & -S_{12} \\
0 & 0 \\
0 & -S_{23} \\
0 & -s_{24}
\end{pmatrix}
\begin{pmatrix}
X_f \\
X_d
\end{pmatrix}
= 
\begin{pmatrix}
S_{11} & 0 & S_{14} & 0 & c_{14} & 0 & 0 & S_{15} \\
-s_{12} & 1 & 0 & 0 & S_{24} & s_{24} & 0 & 0 & S_{25} \\
-s_{13} & 0 & S_{34} & -1 & S_{34} & s_{34} & 0 & 0 & S_{35} \\
S_{14} & s_{14} & 0 & S_{44} & s_{44} & -1 & 0 & S_{45} \\
c_{14} & 0 & s_{14} & 0 & -s_{44} & 0 & 0 & -1 & s_{45}
\end{pmatrix}
\begin{pmatrix}
Z_f \\
Z_d \\
D_f \\
D_d \\
T_f \\
T_d \\
t_2 \\
t_1 \\
t_1^{T} \\
t_2^{T}
\end{pmatrix}
\]

This equation has lost the structural 1’s and 0’s of the standard form. To recover this standard form, some row manipulations have to be done. It is this seventh column of \(S\) which indicates the elements in causal relation with the switch.

- If \(s_{24} \neq 0\), one element recovers its integral causality (one element leaves \(X_d\) for \(X_f\));
- if \(s_{24} = 0\) and \(s_{14} \neq 0\), a resistive element exchanges its causality with the switch;
• if \( s_{44} = 0, s_{s4} = 0 \) and \( e_{44} \neq 0 \), one element looses its integral causality (one element leaves \( X_i \) for \( X_d \));

• the situation \( s_{44} = 0, s_{s4} = 0 \) and \( e_{44} = 0 \), corresponds to a none acceptable configuration of the switches and if \( s_{44} \neq 0 \), there is a solution only if another switch commutates at the same time.

6.1. Worked case when one element loses its integral causality

Let's consider the case of a system where one element loses its integral causality on commutation. Let's call \( x_{id} \) the power variable associated to this element. Reordering the vectors and matrices, we can write :

\[
\begin{bmatrix}
1 & 0 & -S_{12} \\
0 & 1 & -S_{12} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & S_{id} \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
X_i \\
S_{id} \\
X_d \\
\end{bmatrix} =
\begin{bmatrix}
S_{13} & S_1^T & 0 & S_{14} & c_{14} & 0 & 0 & S_{15} \\
-s_{1} & 0 & 0 & S_{13} & 0 & I_{14} & e_{14} & 0 & 0 & S_{15} \\
-s_{1} & -s_{1}^T & -I & 0 & 0 & S_{24} & 0 & 0 & 0 & S_{25} \\
-s_{1} & -s_{1}^T & -s_{1} & S_{33} & -I & S_{34} & 0 & 0 & 0 & S_{35} \\
S_{14} & I_{14} & 0 & S_{14} & 0 & S_{44} & s_{44} & -I & 0 & S_{45} \\
0 & 0 & 0 & -s_{44} & 0 & 0 & -1 & s_{45} \\
\end{bmatrix}
\begin{bmatrix}
Z_i \\
Z_d \\
D_1 \\
D_2 \\
T_i \\
T_o \\
T_1 \\
T_2 \\
U \\
\end{bmatrix}
\]

It is this seventh column of \( S \) which indicates the elements in causal relation with the switch. If the third line were not zero, one element would have recovered its integral causality; if the third line was zero and the fourth line was not zero, a resistive element would have exchanged its causality with the switch. Since these two elements are null, and \( e_{14} \neq 0 \), the corresponding element gets the derivative causality on commutation.

As we have supposed that it is the last element of \( X_i \) which loses its integral causality, and if we decide to put this element on top of \( X_d \), then after exchanging \( I_1 \) and \( I_2 \) we obtain :
which has lost the structural 1’s and 0’s of the standard form. If we multiply each side of the preceding relation by the nonsingular matrix:

\[
\begin{pmatrix}
1 & 0 & -s_{13} \\
0 & 1 & -s_{12} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -s_{24} \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
x
s_d
\end{pmatrix}
= \begin{pmatrix}
S_{16} & s_{16}^T & 0 & S_{13} & 0 & S_{14} & 0 & 0 & c_{14} & S_{15} \\
-1 & 0 & 0 & s_{23} & 0 & t_{14} & 0 & 0 & e_{14} & S_{25} \\
-1 & -s_{12}^T & -s_{13} & 0 & S_{33} & 0 & 0 & 0 & 0 & S_{35} \\
S_{24} & t_{14}^T & 0 & S_{34} & 0 & S_{44} & 0 & -1 & s_{44} & S_{45} \\
e_{14} & 0 & 0 & 0 & 0 & -s_{44} & -1 & 0 & 0 & S_{45}
\end{pmatrix}
\begin{pmatrix}
x_d \\
s_d \\
Z \\
T \\
U
\end{pmatrix}
\]  (11)

where \( V^{-1} \) is directly composed with columns in (11) : column 1 from the matrix on the left part and columns 2,3,5, 8 and 9 from the matrix on the right part, we obtain a new expression which has recovered the standard implicit form.

As the structural relation (which is not unique) depends only on the switches configuration, but not on the way to reach this configuration, the case of \( N \) switches commutating at the same time, can be solved as \( N \) consecutive commutations of one (or two) switch(es).

7. Example

In order to illustrate these results, let us consider first a simple example of an electric car on a slope (Figure 6) driven by a DC motor coupled by an ideal clutch. When the motor is coupled, the velocities of the car and the motor are dependant and when it is decoupled, no torque is transmitted to the car.
7.1. The bond graph

The bond graph model is shown on figure 7. It has the following elements:
- \( I_6 \): inertia (mass of the car),
- \( R_5 \): viscous friction (wind),
- \( S\ell_7 \): gravity force \(( -I_6 \ell g \sin(\Theta) )\) on the car,
- \( S\ell_f \): input current in the motor,
- \( I_2 \): the rotating inertia of the motor,
- \( R_3 \): viscous friction in rotation,
- \( TF \): rotation-translation transformation ratio (\(n\)).
- \( G\ell \): electro-mechanical conversion of the motor (gyrator with ratio \(k\)).

The clutch is represented by the T element. Flows \( f_2 \) and \( f_6 \) respectively represent the angular speed of the motor and the linear speed of the car.

If the clutch is perfect, when it is let out, the motor and the car are not coupled (mode 1). In this case, the clutch can be modeled by a zero effort source and no torque is transmitted on the shaft. When the clutch is let in, the motor and the car are coupled (mode 2), the speed of the car and of the motor are algebraically constrained.
The clutch can be modeled as a zero flow source. Let’s consider mode 2 as the reference mode. During the assignment procedure, when we apply the necessary causality to the bond 4 and extending the causal implication through the 0-junction and the 1-junctions, one of the I elements must have the derivative causality.

\[ S_f : E \xrightarrow{1} GY \xrightarrow{1} 0 \xrightarrow{T} TF \xrightarrow{1} 1 \xrightarrow{I} \]

\[ \xrightarrow{4} \quad \xrightarrow{2} \quad \xrightarrow{6} \quad \xrightarrow{7} \quad Se \]

\[ R \quad \xrightarrow{3} \quad R \]

**Figure 8. Causal bond graph when the motors are coupled (mode 2)**

### 7.2. The implicit junction structure equation

For this causal bond graph, the implicit equation is:

\[
\begin{bmatrix}
1 & n \\
0 & 0 \\
0 & 0 \\
0 & -n
\end{bmatrix}
\begin{bmatrix}
P_n \\
P_0 \\
P_4 \\
P_6
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & -1 & -n & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
f_2 \\
f_5 \\
f_3 \\
f_4 \\
ed_s \\
Se_1 \\
Se_2
\end{bmatrix}
\]

(12)

After a commutation, consisting in exchanging columns 7 and 8, the standard implicit form can be recovered by multiplying (12) by a square invertible matrix constituted with columns 1 and 2 from the left hand side and columns 5, 6 and 7 from the right hand side of equation (12):

\[
\begin{bmatrix}
1 & n \\
0 & 0 \\
0 & 0 \\
0 & -n
\end{bmatrix}^{-1}
= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The implicit equation for the other mode is then:
which is the standard implicit junction structure relation in mode 1.

7.3. State equations

An implicit general state equation such as (4) can be deduced as follows:

\[
\begin{bmatrix}
1 & n & 0 & 0 & \dot{p}_2 \\
0 & 0 & 0 & 0 & \dot{p}_b \\
0 & -n & 0 & 0 & \dot{f}_4 \\
0 & 0 & 0 & 0 & \dot{e}_4
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{n} & \left( \frac{R_3 + n^2 R_s}{I_2} \right) & 0 & -n^2 R_s & 0 & k & n \\
0 & \frac{n}{I_2} & 0 & -1/I_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-x & x & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_2 \\
p_b \\
f_4 \\
e_4 \\
Sf_1 \\
Se_7
\end{bmatrix}
\]

This state equation is valid for both configurations.

However, more simple distinct state equations can be found for each configuration. The first one, when the motors are coupled \( x = 0 \), so that \( f_4 = 0 \), is directly derived by using the first 2 lines:

\[
\begin{bmatrix}
1 & n & 0 & 0 & \dot{p}_2 \\
0 & 0 & 0 & 0 & \dot{p}_b \\
0 & -n & 0 & 0 & \dot{f}_4 \\
0 & 0 & 0 & 0 & \dot{e}_4
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{n} & \left( \frac{R_3 + n^2 R_s}{I_2} \right) & 0 & -n^2 R_s & 0 & k & n \\
0 & \frac{n}{I_2} & 0 & -1/I_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-x & x & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_2 \\
p_b \\
f_4 \\
e_4 \\
Sf_1 \\
Se_7
\end{bmatrix}
\]

When the motors are coupled, \( x = 1 \) so that \( e_4 = 0 \); to eliminate \( f_4 \), we need to combine the first three lines, and we get:

\[
\begin{bmatrix}
1 & 0 & 0 & \dot{p}_2 \\
0 & 1 & 0 & \dot{p}_b \\
0 & 0 & 0 & \dot{f}_4 \\
0 & 0 & 0 & 0 & \dot{e}_4
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{n} & \left( \frac{R_3 + n^2 R_s}{I_2} \right) & 0 & -n^2 R_s & 0 & k & n \\
0 & \frac{n}{I_2} & 0 & -1/I_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-x & x & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_2 \\
p_b \\
f_4 \\
e_4 \\
Sf_1 \\
Se_7
\end{bmatrix}
\]
which could have been derived more easily from the standard implicit form in mode 1.

7.4. Computing discontinuities on commutation:

Using the general formulas, we can compute the discontinuities occurring on the transition from mode 1 to mode 2 by applying (9):

\[
\begin{pmatrix}
1 & n \\
0 & -n
\end{pmatrix}
\begin{pmatrix}
p_z^+ - p_2^- \\
p_6^+ - p_6^-
\end{pmatrix}
= \begin{pmatrix} 0 \\ -T \end{pmatrix}
\]

and (10):

\[
n \frac{p_2^+}{I_2^+} - \frac{p_6^+}{I_6^+} = 0
\]

which finally gives:

- the amplitude of the effort pulse in the clutch:

\[
T = n \frac{n I_6^+ p_2^- - I_2^- p_6^-}{(I_2^+ + n I_6^+)}
\]

- the variables after the commutation:

\[
p_z^+ = p_2^- - T \\
p_6^+ = p_6^- + T/n
\]

These values satisfy the principle of the conservation of momenta.

8. Application to the simulation of complex hybrid system

The results developed in the preceding sections can be used to simulate complex hybrid systems.

8.1. Description of the method

The first step of the method consists in deriving the standard implicit form in one mode from the causal graph of the system, as described in section 3. Using the results of section 6 the standard implicit form can be computed for each mode.

Section 5 allows to define the implicit equation with index 1 for each mode. This equation can be handled by standard DAE (differential algebraic equations) solvers. In our case, DASSL [BRE 89] has been used to simulate each implicit equation between each commutation. The discontinuities and pulses at the commutation are computed using results of section (5), and the global simulation scheme is depicted on figure 9.
According to the current mode, which is defined by the discrete-event automata, a model is selected and initial conditions are computed. Internal events such as the zero crossing of current or voltage in electrical devices as diodes or thyristors are generated from the continuous variables.

8.2. The example

The example consist of an asynchronous machine with its starter driving a load [BUI 96] (figure 10). E1, E2, E3 is a three-phase voltage in star. The switching components are gradators, each of which is composed of two thyristors. The discrete part corresponds to the control of the three switches. These switches can be closed when the voltage is positive, and they fall in the open state when their current becomes zero. The control law consists in modulating a delay angle : at $t=0$ the delay angle is equal to $2\pi/3$ and diminishes linearly to 0 at $t=0.2$ s.

8.2.1. Determination of the acausal bond graph

The electrical equations of the system are the following :
at the stator

\[
\begin{align*}
V_{as} &= -\frac{d\Psi_{as}}{dt} - R_{as}i_{as} \\
V_{bs} &= -\frac{d\Psi_{bs}}{dt} - R_{bs}i_{bs} \\
V_{cs} &= -\frac{d\Psi_{cs}}{dt} - R_{cs}i_{cs}
\end{align*}
\]

at the rotor

\[
\begin{align*}
V_{ar} &= \frac{d\Psi_{ar}}{dt} + R_{ar}i_{ar} = 0 \\
V_{br} &= \frac{d\Psi_{br}}{dt} + R_{br}i_{br} = 0 \\
V_{cr} &= \frac{d\Psi_{cr}}{dt} + R_{cr}i_{cr} = 0
\end{align*}
\]

The magnetic relation between \( \Psi = (\Psi_{as}, \Psi_{bs}, \Psi_{cs}, \Psi_{ar}, \Psi_{br}, \Psi_{cr})^T \) and \( i = (i_{as}, i_{bs}, i_{cs}, i_{ar}, i_{br}, i_{cr})^T \) is \( \Psi(\Theta, i) = L(\Theta) i \) with \( L(\Theta) \) defined by:

\[
\begin{bmatrix}
L_{as} & M_{as} & M_{as2} & M_{as1, \text{cod}(b)} & M_{as12, \text{cod}(b)} & M_{as13, \text{cod}(b)} \\
M_{as} & L_{as2} & M_{as3} & M_{as21, \text{cod}(b)} + \frac{2\pi}{7} & M_{as22, \text{cod}(b)} & M_{as23, \text{cod}(b)} + \frac{4\pi}{7} \\
M_{as2} & L_{as3} & M_{as3} & M_{as31, \text{cod}(b)} + \frac{2\pi}{7} & M_{as32, \text{cod}(b)} & M_{as33, \text{cod}(b)} + \frac{4\pi}{7} \\
M_{as1, \text{cod}(b)} & M_{as21, \text{cod}(b)} + \frac{2\pi}{7} & M_{as12, \text{cod}(b)} & L_{as1} & M_{as1} & M_{as2} \\
M_{as21, \text{cod}(b)} + \frac{2\pi}{7} & M_{as22, \text{cod}(b)} & M_{as23, \text{cod}(b)} & L_{as2} & M_{as2} & M_{as3} \\
M_{as31, \text{cod}(b)} + \frac{2\pi}{7} & M_{as32, \text{cod}(b)} & M_{as33, \text{cod}(b)} & M_{as3} & L_{as3} & L_{as3}
\end{bmatrix}
\]

From an electrical point of view, the motor is an inertial element (there is a relation between the flow and the integral of the effort \( \Psi(\Theta, i) = L(\Theta) i \), but the value of the inertia \( L \) is a function of the position. The motor torque is given by the following relation [ROS 83]:

\[
C_m(\Theta, i) = -\frac{1}{2} i^2 \frac{\partial L}{\partial \Theta} i.
\]

As the torque is a function of the angular position, from a mechanical point of view the motor is a capacitive element; therefore, it is an IC multiport.

The acausal bond graph of Figure 11 can be deduced from the above equations where the mechanical load consists of an inertia associated to a friction.
For this system, five modes are possible: - the 3 switches closed (mode 0), - 2 switches closed, one open (mode 1, mode 2, mode 3), - at least 2 switches open (mode 4).

8.2.2. The implicit standard forms and the implicit state forms

According to the method, the causality is assigned to the bond graph in one mode in order to determine the implicit standard form. The causal bond graph in the mode 0 is presented on Figure 12. The 3 switches are closed so that they are equivalent to 0 effort sources.

8.2.3. Simulation results. Comparison with classical methods

Figure 13 presents the evolution of some important variables.
The usual approaches to simulate such a system use a constant topology of the circuit with non linear elements to model the switches (for instance a resistance with a small value or a large value according to the state). This results in a more simple modeling approach since there is just one model with non linear elements. But this apparent simplicity leads to problems in the simulation: what value should be given to these resistances? This will generally lead to stiff differential equation, with the necessity to modify the simulation step on commutation. Moreover, it can lead to wrong interpretation of the results: the amplitude of the pulses on commutation is a function of the value of the resistive elements which have not been chosen on the basis of physical consideration, but from a numerical simulation point of view.

The approach proposed here can seem more complicated because it needs to elaborate several models and to compute discontinuities functions. But those computations can be automated, and the resulting simulation is numerically much easier because no arbitrary parameters are required to be defined.

Conclusion

This paper has shown some properties concerning the models of the continuous part of hybrid linear physical systems using the notion of implicit junction structure relation:

- A unique implicit state equation with jumping parameters has been proposed.
• When using a different equation for each configuration, it has been shown that the system can be modeled by non-impulsive implicit linear state equations.
• An explicit solution has been proposed to compute variables on the commutation.
• A solution has been proposed to compute the implicit junction structure and then the implicit state equation for each mode.

Those last two points offer an alternative for the simulation of hybrid systems. Rather than using non-linear elements to model switches, which can lead to numerical difficulties, it is easier to use ideal components, with some simple extra formal computing to deduce the different models, and to change the model on commutation. This method has proved to be efficient in the simulation of a large system consisting of an asynchronous motor associated to a starter.

References


[LOR 93] LORENZ I. F., "Discontinuities in Bond graphs : what is required ?", ICBGM'93, SCS Western Multiconference, San Diego, January 17-20, 1993

